Solution to Excercise 3.12
Let (S, d) be a metric space.

1. Show that if E is a closed subset of a sequentially compact subset F of S, then E is sequentially compact.
   Proof: Let (s) be a sequence in E. Then (s) is also a sequence in F and hence there is a subsequence (t) of (s) which converges to some element x of F. Since all the terms of (t) lie in E and E is closed, $x \in E$. We have thus shown that every sequence in E has a subsequence which converges to an element of E.

2. Show that a finite union of sequentially compact sets is compact.
   Proof: Let $F_1, \ldots F_k$ be a finite number of sequentially compact subsets of S and let F be their union. Suppose that (s) is a sequence in F. Since there are only finitely many $F_1, \cdots F_k$, at least one of them must contain s_{n_i} for infinitely many i. Thus there is a subsequence t of (s) which is contained in one of the F_j's. Since this set is sequentially compact, this subsequence has a subsequence which converges to an element of F_j, and is a subsequence of the original sequence which converges to an element of F.