Exercises on Log Geometry

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1. Let k be an algebraicaly closed field, let P be a fine monoid, and let $\mathsf{A_P}$ denote the log scheme associated to $P \to k[P]$. Show that if X is any log scheme, there is a natural bijection
$$\mathrm{Mor}(X, \mathsf{A_P}) \cong \mathrm{Hom}(P, \Gamma(X, \mathcal{M}_X)).$$

2. Recall that a homomorphism of integral monoids $\theta\colon P \to Q$ is *locally exact* if for every face G of Q, the homomorphism $P_F \to Q_G$ is exact, where $F := \theta^{-1}(G)$.
   (a) Give an example of a locally exact homomorphism which is not exact.
   (b) Give an example of an exact homomorphism which is exact but not locally exact.
   (c) Give an example of a homorophism which is locally exact but not integral.

3. If Q is a fine monoid, we let R_Q denote the set of homomorphisms from the monoid Q to the multiplicative monoid of nonnegative real numbers and let $C(Q)$ denote the real cone generated by Q. Recall that if S is finite set of generators for $C(Q)$ the moment map
$$\mu_s\colon R_Q \to C(Q) : \rho \mapsto \sum_{s \in S} \rho(s)s$$
is a homeomorphism. Prove this by hand in the following cases:
   (a) $Q = \mathbf{N}$ and S is any nonempty finite subset of $\mathbf{N}^+$.
   (b) $Q = \mathbf{Z}$ and S is any finite subset of $\mathbf{Z}$ containing at least one positive and one negative number.