p-adic Ordinals of Eigenvalues of F-crystals

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November 21, 2018

On page 8-46 of [1], I gave an incorrect definition of the slopes of an F-crystal. Namely, I asserted that if (E, Φ) is an F crystal over the Witt ring W of a perfect field k, then its slopes can be computed by writing the matrix for Φ in any basis and computing the p-adic ordinals of the eigenvalues of this matrix. This is false, and in fact these ordinals are not independent of the choice of basis.

Here is an example. Let W be the Witt ring of the field with 9 elements, which can be expressed as $\mathbf{Z}_3[i]$, where $i^2 = -1$. Let E be the rank two F-crystal over W given in a basis x, y by $\Phi(x) = y$, $\Phi(y) = 3x$. The eigenvalues of the linear map defined by these equations are $\pm i\sqrt{3}$. Now consider the basis x, y' for this crystal, where $y' := y + ix$ and $y = y' - ix$. Then $F_W(i) = -i$, and so

$$\Phi(x) = y = y' - ix$$

and

$$\Phi(y') = \Phi(y + ix) = \Phi(y) - i\Phi(x) = 3x - i(y' - ix) = 2x - iy'.$$

The corresponding linear map is given by the matrix $\begin{pmatrix} -i & 2 \\ 1 & -i \end{pmatrix}$. One sees from the Newton polygon of its characteristic polynomial $t^2 + 2i - 3$ that the ordinals of its eigenvalues are 0 and 1. (In fact these eigenvalues are $-i \pm \sqrt{2}$.)

References

[1] P. Berthelot and A. Ogus. *Notes on Crystalline Cohomology*, volume 21 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, 1978.