Matrices and Linear Transformations

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Functions and mappings

- Let S and T be sets.
- A mapping or function from S to T is a rule which assigns to every element of S a (well-defined) element of T.
- The set S is called the source or domain of f. The set T is called the target or codomain of f.
- If s is an element of S, the value (output) of f at s is denoted by f(s).

Notation

- $f : S \to T$
- $s \mapsto f(s)$
- $t = f(s)$

Inverse functions

- A function $f: S \to T$ is said to be <u>invertible</u> if for every t in T there is a unique s in S such that $f(s) = t$.
- Then there is a (unique) function $g: T \to S$ such that $g(t)$ is the s such that $f(s) = t$.
- This g is called the <u>inverse</u> of the function f.

Linear transformations

- Let A be an n x m matrix.
- Define a function $T_A: R^m \rightarrow R^n$ by the rule: $X \mapsto AX$ (matrix multiplication).
- A function $T: R^m \rightarrow R^n$ is said to be a linear transformation if there exists a matrix A such that $T = T_A$.

The inverse of a linear transformation

- **Theorem**: Let A be an n x m matrix. Then T_A is invertible if and only if n = m = rank(A).
- We will discuss how to compute the inverse later.

Key formula:

- The <u>standard frame</u> for R^m is the sequence of vectors $(e_1, e_2, \ldots\ldots, e_m)$, where e_j is the vector with 1 in the j^{th} place and zeroes everywhere else.
- Then if A is an n x m matrix, $T_A(e_j)$ is the j^{th} column of A:
- $T_A(e_j) = C_j(A)$.

Drawing matrices

- One way to draw a matrix is just to draw its columns.
- This shows the effect of T_A on the standard frame.