The trace map for projective space

For simplicity we work over an affine scheme $S = \text{Spec } R$. Recall that if $E$ is an $R$-module, then $PE$ is the scheme $\text{Proj } S[E]$. On $PE$ there is a canonical exact sequence of quasi-coherent sheaves:

$$0 \to \mathcal{H} \to E \otimes O_{PE} \to O_{PE}(1) \to 0.$$  

Here we have written $E \otimes O_{PE}$ to mean $\pi^* \tilde{E}$, where $\tilde{E}$ is the quasi-coherent sheaf on $S$ associated to the $R$-module $E$. The map $u: E \otimes O_{PE} \to O_{PE}(1)$ is the universal invertible quotient of $E$, and $\mathcal{H}$ is the universal hyperplane in $E$. Tensoring the above sequence with $O_{PE}(-1)$, we find:

$$0 \to \mathcal{H}(-1) \to E \otimes O_{PE}(-1) \to O_{PE} \to 0.$$  

We shall use the fact that there is a canonical isomorphism

$$\mathcal{H}(-1) \cong \Omega^1_{PE/R}.$$  

Now suppose that $E$ is projective of rank $n+1$. Then the isomorphism above induces an isomorphism:

$$\Lambda^{n+1} E(-n-1) \cong \Omega^n_{PE/S}.$$  

The Koszul complex of the homomorphism $E \otimes O_{PE}(-1) \to O_{PE} \to 0$ is the complex:

$$0 \to \Lambda^{n+1} E(-n-1) \to \Lambda^n E(-n) \to \cdots \to E(-1) \to O_{PE} \to 0.$$  

It is exact because the mapping $u$ is surjective. Thus the complex:

$$K^- := \Lambda^n E(-n) \to \cdots \to E(-1) \to O_{PE}$$

is an $n$-term resolution of the complex $\Lambda^{n+1} E(-n-1) \cong \Omega^n_{PE/R}$. By the projection formula,

$$H^q(PE, K^j) = \Lambda^{n-j} E \otimes H^q(PE, O_{PE}(j-n)).$$  

Since $j-n > -n-1$, these groups vanish if $q > 0$, so the complex $K^-$ is in fact an acyclic resolution of $\Omega^n_{PE/R}$. Hence the complex of global sections $H^0(PE, K^-)$ calculates the cohomology of $\Omega^n_{PE/R}$. But this complex vanishes except in degree $n$, where it is canonically isomorphic to $R$. We deduce a canonical (and coordinate free) isomorphism

$$H^n(PE, \Omega^n_{PE/R}) \cong R.$$