1. Let $p$ be an odd prime, and let $p^* := (-1)^{\frac{p-1}{2}}$. Recall that $K := \mathbb{Q} (\sqrt{p^*})$ is the unique subfield of $\mathbb{Q} (\zeta_p)$ which has degree 2 over $\mathbb{Q}$. It follows that $K$ is unramified over $\mathbb{Q}$ away from $p$. Recall that if $q$ is an odd prime, we computed $\left( \frac{q}{p} \right)$ by looking at the Frobenius element of $\text{Gal}(K/\mathbb{Q})$ at $q$. Use the same method to show that

$$\left( \frac{2}{p} \right) = (-1)^{\frac{p^2 - 1}{8}}.$$

Hint: Compute the integral closure of $\mathbb{Z}$ in $K$ at 2.

2. Nuekirch, §11, number 1.

3. Nuekirch, §11, number 2

4. Nuekirch, §11, number 3