1. Let $k$ be a field, let $\overline{k}$ be an algebraic closure of $k$, and let $A$ be a finite dimensional $k$–algebra. If $a \in A$, recall that $Nm_{A/k}(a)$ is by definition the determinant of the $k$-linear endomorphism $a_A: A \to A$ (multiplication by $a$). Let $S(\overline{k})$ be the set of $k$-homomorphisms from $A \to \overline{k}$ (the “geometric points” of $\text{Spec } A$ in $\overline{k}$). Find and prove a formula for the image of $Nm_{A/k}(a)$ in $\overline{k}$ in terms of $S(\overline{k})$ and some “multiplicities” attached to each $\sigma \in S(\overline{k})$.

2. In the situation above, suppose that $B/A$ is a finite and projective $A$-algebra. Can you prove that $Nm_{A/k}Nm_{B/A} = Nm_{B/k}$ using your formula? Can you prove this assuming that $A/k$ or $B/k$ is separable? Can you prove it without assuming that $k$ is a field?

3. Neukirch, page 15: 1–3

4. Neukirch, page 23: 1, 2, 3, 4, 9, 10