1. Let Ab denote the category of abelian groups and let D be the contravariant functor from Ab to Ab which takes A to $\mathrm{Hom}(A, \mathbf{Z})$. (What does it do to homomorphisms?) Prove that the evaluation mapping $\epsilon\colon A \to DD(A)$ defines a natural transformation from the identity functor to DD.

2. Let M be a monoid and let Ens_M denote the category of M-sets. Let $F\colon Ens_M \to Ens$ denote the forgetful functor.

   - Show that for $m \in M$, left multiplication by m defines a natural transformation $\lambda(m)\colon F \to F$. Show that the map $\lambda\colon M \to \mathrm{End}(F)$ is an isomorphism of monoids.
   - Show that m belongs to the center of M, if and only if $\lambda(m)$ defines a natural transformation $I \to I$, where $I\colon Ens_M \to Ens_M$ is the identity functor

3. (Yoneda). Let C be a category and let X be an object of C. Recall that if $T \in Ob(C)$, then $h_X(T) := Arr_C(T, X)$, and if $\theta \in Arr_C(T, T')$, then $h_X(\theta)\colon h_X(T') \to h_X(T)$ sends h to $h \circ \theta$.

   (a) Verify that h_X is a contravariant functor from C to Ens.

   (b) Suppose that F is any contravariant functor from C to Ens. If $\xi \in F(X)$ and $\theta \in h_X(T)$, let $\hat{\xi}_T(\theta) := F(\theta)(\xi)$. Thus $\hat{\xi}_T\colon h_X(T) \to F(T)$. Show that $\hat{\xi}$ defines a natural transformation $h_X \to F$.

   (c) Show that $\xi \to \hat{\xi}$ induces a bijection from $F(X)$ to the class of natural transformations $h_X \to F$. (Hint: To define a map in the other direction: If η is such a natural transformation, look at $\eta_X(\mathrm{id}_X) \in F(X)$.)

   (d) Show that Cayley's theorem for monoids is a special case and weak version of Yoneda's theorem, by interpreting Yoneda's theorem as follows. View a monoid M as a category with one object p, so that $M = End(p)$. Show that a covariant functor from M to Ens amounts to a left M-set, and that a contravariant functor from M to Ens amounts to a right M-set. In particular, what is h_p as a right M-set? What are the natural transformations $h_p \to h_p$? What does Yoneda say and how does it imply Cayley?