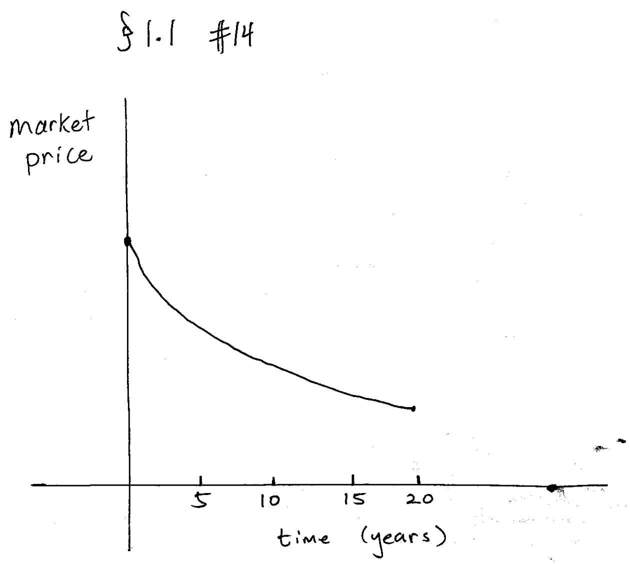
**Math 1A — UCB, Fall 2010 — A. Ogus**
**Solutions[1] for Problem Set 1**

§1.1 # 6. Determine whether the curve is a graph of a function of $x$. If it is, state the domain and range of the function.
**The curve is a function since it passes the vertical line test. The domain is $[-2, 2]$. The range is $[-1, 2]$.**

□

§1.1 # 14. Sketch a rough graph of the market value of a new car as a function of time for a period of 20 years. Assume the car is well maintained.
**Sketches will vary. The graph should start high and then go down over time. Here is a possible example:**

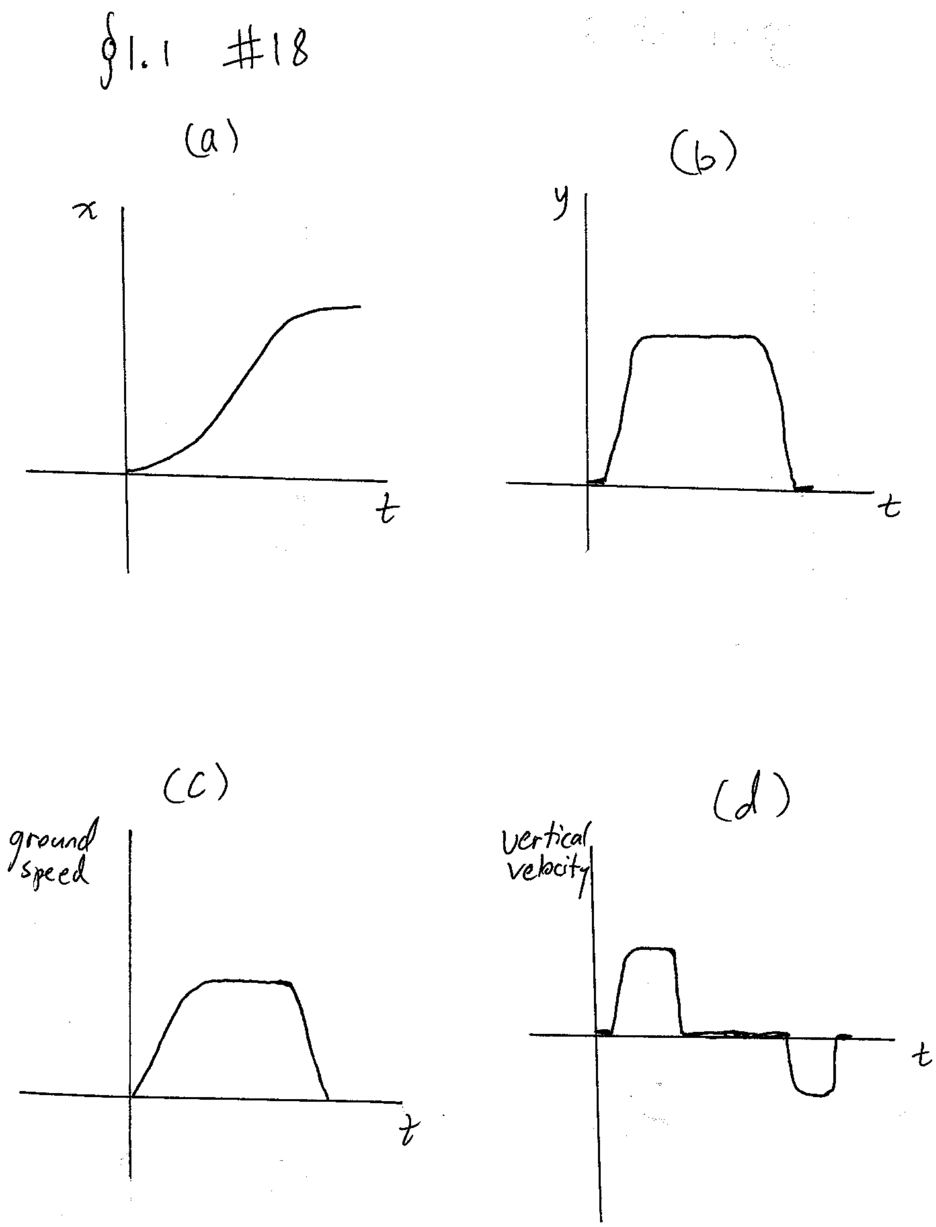



□

§1.1 # 18. An airplane takes off from an airport and lands an hour later at another airport, 400 miles away. If $t$ represents the time in minutes since the plane has left the terminal building, let $x(t)$ be the horizontal distance traveled and $y(t)$ be the altitude of the plane.

(a) Sketch a possible graph of $x(t)$.
(b) Sketch a possible graph of $y(t)$.
(c) Sketch a possible graph of the ground speed.
(d) Sketch a possible graph of the vertical velocity.

**Sketches again will vary somewhat.**




□

§1.1 # 30. Find the domain of the function $g(u) = \sqrt{u} + \sqrt{4-u}$.
**The only constraint for $g(u) = \sqrt{u} + \sqrt{4-u}$ is that what's under the square root sign must be non-negative. Therefore, the domain is $\{u\colon u \geq 0 \,\text{and}\, 4-u \geq 0\}$. The condition "$u \geq 0 \,\text{and}\, 4-u \geq 0$" is equivalent to "$4 \geq u \geq 0$", so the domain may be written more simply as $\{u\colon 4 \geq u \geq 0\}$ or just $[0,4]$.**

□

§1.1 # 57. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side $x$ at each corner and then folding up the sides as in the figure. Express the volume $V$ of the box as a function of $x$.
**The length of the box thus formed is $20-2x$, the width $12-2x$, and the height $x$. Since these three dimensions should all be positive numbers, we see that $x$ should be greater than 0 and less than 6. I.e., the domain of $V$ is $\{x\colon 0 < x < 6\}$. The volume $V$ of the box is then given by multiplying the length, width, and height:**

$$\begin{aligned} V(x) &= (\text{length}) \cdot (\text{width}) \cdot (\text{height}) \\ &= (20-2x)(12-2x)x \\ &= (240 - 24x - 40x + 4x^2)x \\ &= 4x^3 - 64x^2 + 240x \end{aligned}$$

□

§1.2 # 4. Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator.)

**(a) $G$, since $y = 3x$ is the graph of a straight line.**
**(b) $f$, since the range of an exponential function such as $y = 3^x$ is the set of all positive real numbers.**
**(c) See the answer to (d) below.**
**(d) The answer to (c) is $F$ and (d) is $g$ since as $x$ gets bigger and bigger, $y$ will get bigger much faster in the case of $y = x^3$ than $y = \sqrt[3]{x}$.**

□

§1.3 # 6. The graph of $y = \sqrt{3x - x^2}$ is given. Use transformations to create a function whose graph is as shown.
**The graph appears to have moved to the right 2 units and stretched vertically by a factor of 2. Therefore, if $f(x) = \sqrt{3x - x^2}$ is the original function, then we need to find $2f(x-2)$, or $2\sqrt{3(x-2) - (x-2)^2}$. Simplifying yields $2\sqrt{3x - 6 - x^2 + 4x - 4}$, and finally $2\sqrt{-x^2 + 7x - 10}$.**

□