Math 1A — UCB, Spring 2010 — A. Ogus
Solutions[1] for Problem Set 10

§4.5 # 5. $y = x^4 + 4x^3$
Solution.
1. Domain : $\mathbb{R}$
2. x-intercepts : $x^4 + 4x^3 = 0$, i.e. $x = 0, x = -4$. y-intercept : $y = 0$.
3. symmetry : no
4. asymptote : no
5. I/D intervals : $y' = 4x^3 + 12x^2$. So, when $x < -3$ it is decreasing but it is increasing otherwise.
6. Local Max/Min : $x = 3$, it has a local minimum value -27.
7. Concavity : $y'' = 12x^2 + 24x$. So, when $x < -2$ or $x > 0$ it is concave upward but it is concave downward otherwise. $x = 0, -2$ is the inflection points. The graph will be attached. □

§4.5 # 15. $y = \frac{x-1}{x^2}$
Solution.
1. Doamain : $x \neq 0$.
2. x-intercepts : $x = 1$, no y-intercept because $x = 0$ is not in the domain.
3. symmetry : no
4. asymptote : $y = 0$ is a horizontal asymptote and $x = 0$ is a vertical asymptote.
5. I/D intervals : $y' = \frac{x^2-(x-1)2x}{x^4} = \frac{2-x}{x^3}$. So, when $0 < x < 2$ it is increasing but it is decreasing otherwise.
6. Local Max/Min : $x = 2$, it has a local maximum value $1/4$.
7. Concavity : $y'' = \frac{-x^3-(2-x)3x^2}{x^6} = \frac{2x-6}{x^4}$. So, when $x > 3$ it is concave upward but it is concave downward otherwise. $x = 3$ is the inflection point. The graph will be attached. □

§4.5 # 21. $y = \sqrt{x^2 + x - 2}$
Solution.
1. Domain : $x^2 + x - 2 \geq 0$, so $x \leq -2$ or $x \geq 1$.
2. x-intercepts : $\sqrt{x^2 + x - 2}$=0, so $x = -2, x = 1$, no y-intercept because $x = 0$ is not in the domain.
3. symmetry : This graph is symmetric with respect to the line $x = -1/2$.
4. asymptote : When x is positive, $y = x\sqrt{1 + 1/x - 2/x^2}$. When x goes to ∞, $\sqrt{1 + 1/x - 2/x^2}$ goes to 1, so this graph looks like $y = x$. But we don't know how much they differ. Taking the limit of $x\sqrt{1 + 1/x - 2/x^2} - x$, we can find their difference. The second form is $\frac{0}{0}$, we can apply l'Hospital's Rule.
$\lim_{x\to\infty} x\sqrt{1 + 1/x - 2/x^2} - x = \lim_{x\to\infty} \frac{\sqrt{1+1/x-2/x^2}-1}{1/x} = \lim_{t\to 0^+} \frac{\sqrt{1+t-2t^2}-1}{t} = \lim_{t\to 0^+} \frac{1-4t}{2\sqrt{1+t-2t^2}} = 1/2$. Therefore the slant asymptote is $y = x + 1/2$ when x goes to ∞. By the symmetry, when x goes to $-\infty$, the slant asymptote is $y = -x - 1/2$.
5. I/D intervals : $y' = \frac{2x+1}{2\sqrt{x^2+x-2}}$, so when $x > -1/2$, this graph is increasing, otherwise it is decreasing.
6. Local Max/Min : The solution of $y' = 0$ is only $x = -1/2$ but this x is not in the domain. So this function does not have any local Max/Min.
7. Concavity : Second derivative of y is $\frac{-9/4}{\sqrt{(x^2+x-2)^3}}$, so it is always negative, i.e. concave downward on the whole domain. The graph will be attached. □

§4.5 # 38. $y = \frac{\sin x}{2+\cos x}$

Solution.
1. Domain : $\mathbb{R}$
2. x-intercepts : $\sin x = 0$, so $x = n\pi$. y-intercept : $y = 0$.
3. symmetry : this is periodic function and its periodicity is 2π. Therefore we draw the graph only for $0 \le x \le 2\pi$.
4. asymptote : no.
5. I/D intervals : $y' = \frac{\cos x(2+\cos x)-\sin x(-\sin x)}{(2+\cos x)^2} = \frac{2\cos x+1}{(2+\cos x)^2}$, so when $2\cos x + 1 < 0$, i.e. $\frac{2\pi}{3} < x < \frac{4\pi}{3}$, $f(x)$ is decreasing, otherwise it is increasing.
6. Local Max/Min : $y' = 0$, so $x = \frac{2\pi}{3}, \frac{4\pi}{3}$ are the critical points. It has a local maximum value $\frac{1}{\sqrt{3}}$ at $x = \frac{2\pi}{3}$, and it has a local minimum value $\frac{-1}{\sqrt{3}}$ at $x = \frac{4\pi}{3}$.
7. Concavity : $y'' = \frac{-2\sin x(2+\cos x)^2-(1+2\cos x)2(2+\cos x)(-\sin x)}{(2+\cos x)^4} = \frac{2\sin x(\cos x-1)}{(2+\cos x)^3}$, so $0 < x < \pi$ it is concave downward, otherwise it is concave upward. $x = 0, \pi, 2\pi$ are the inflection points. The graph will be attached. □

§4.5 # 52. $y = \arctan\left(\frac{x-1}{x+1}\right)$

Solution.
1. Domain : $x \ne -1$.
2. x-intercepts : $x = 1$, y-intercepts : $y = \arctan(-1) = -\frac{\pi}{4}$
3. symmetry : no.
4. asymptote : When x goes to ∞, $\frac{x-1}{x+1}$ goes to 1, so this function goes to $\frac{\pi}{4}$, i.e. $\frac{\pi}{4}$ is the horizontal asymptote. When x goes to $-\infty$ the same thing happens.
5. I/D intervals : $y' = \frac{2/(x+1)^2}{1+(x-1/x+1)^2} = \frac{1}{(x^2+1)}$, therefore this function is always increasing.
6. Local Max/Min : no.
7. Concavity : $y'' = \frac{2x}{(x^2+1)^2}$, so when x is positive, it is concave upward, otherwise it is concave downward. $x = 0$ is the inflection point. The graph will be attached. □

§4.5 # 56. $F(x) = -\frac{k}{x^2} + \frac{k}{(x-2)^2}$.

Solution.
1. Domain : $0 < x < 2$
2. x-intercepts : $x = 1$, y-intercept : no.
3. symmmetry : It is symmetric with respect to the point (1, 0)
4. asymptote : $x = 0$, $x = 2$ are the vertical asymptote.
5. I/D intervals : $y' = \frac{2k}{x^3} - \frac{2k}{(x-2)^3}$ is always positive, so this function is increasing.
6. Local Max/Min : no.
7. Concavity : $y'' = \frac{-6k}{x^4} + \frac{6k}{(x-2)^4}$, so when $x < 1$ this graph is concave downward, otherwise it is concave upward. $x = 1$ is the inflection point. The graph will be attached. □

§4.5 # 64. $y = e^x - x$

Solution. When x goes to $-\infty$, e^x goes to 0. So, $y = -x$ is the slant asymptote when x goes to $-\infty$. But it does not have slant asymptote when x goes to ∞. □

§4.5 # 68. $y = \sqrt{x^2 + 4x}$

Solution. When x is positive, $y = x\sqrt{1 + 4/x}$, so it looks like $y = x$.(Because $\sqrt{1 + 4/x}$ goes to 1) Find their difference, we will use l'Hospital's Rule.(4th equality)

$\lim_{x\to\infty}(x\sqrt{1+4/x}-x) = \lim_{x\to\infty}\frac{\sqrt{1+4/x}-1}{1/x} = \lim_{t\to 0^+}\frac{\sqrt{1+4t}-1}{t} = \lim_{t\to 0^+}\frac{4}{2\sqrt{1+4t}} = 2$. Therefore $y = x+2$ is the slant asymptote when x goes to ∞. When x is negative, $y = -x\sqrt{1+4/x}$, so its slant asymptote is $y = -x-2$. □

§4.7 # 4. Find a positive number such that the sum of the number and its reciprocal is as small as possible.

Solution. Let $S(x) = x + \frac{1}{x}$ for all $x > 0$. We wish to find the absolute minimum of S. As the derivative, $S'(x) = 1 - \frac{1}{x^2}$, is well-defined on its domain ($x > 0$) and equals zero only when $x = 1$, we see that $x = 1$ is the only critical number of S. To show that $x = 1$ does indeed give an absolute minimum, we shall apply the First Derivative Test for Absolute Extreme Values (p. 324). Consider the sign of S'. If $0 < x < 1$, then $1 < \frac{1}{x^2}$, and so $S'(x) = 1 - \frac{1}{x^2} < 0$. If $1 < x$, then $1 > \frac{1}{x^2}$, and so $S'(x) = 1 - \frac{1}{x^2} > 0$. Noting that S is continuous on its interval domain $(0, \infty)$ — indeed, it is differentiable on this interval — it follows from part (b) of the First Derivative Test for Absolute Extreme Values that $S(1)$ is the absolute minimum value of S. Thus, $x = 1$ is the desired positive number. □

§4.7 # 16(a). Show that of all the rectangles with a given area, the one with smallest perimeter is a square.

Solution. Let the dimensions of our rectangle be x and y. We may vary the dimensions, but the product $A = xy$ must be constant. The perimeter of such a rectangle is $P = 2x + 2y$. Our task is to find the absolute minimum of P and show that in this case $x = y$. But as A is constant, we may substitute $\frac{A}{x}$ for y to obtain the one-variable function $P(x) = 2x + 2\frac{A}{x}$ with domain $x > 0$. The derivative of P is $2\left(1 - \frac{A}{x^2}\right)$, and so it's clear that the only critical number of P is $\sqrt{A}$. As in the solution to problem 4 above, we may apply the First Derivative Test for Absolute Extreme Values. When $0 < x < \sqrt{A}$, it follows that $P'(x) < 0$. And when $x > \sqrt{A}$, it follows that $P'(x) > 0$. Thus, P has an absolute minimum when $x = \sqrt{A}$. We see that in this case $y = \frac{A}{\sqrt{A}} = \sqrt{A}$ as well, so the perimeter is indeed minimized when the rectangle is a square. □

§4.7 # 42. For a fish swimming at a speed v relative to the water, the energy expenditure per unit time is proportional to v^3. It is believed that migrating fish try to minimize the total energy required to swim a fixed distance. If the fish are swimming against a current u ($u < v$), then the time required to swim a distance L is $L/(v-u)$ and the total energy E required to swim the distance is given by

$$E(v) = av^3 \cdot \frac{L}{v-u}$$

where a is the proportionality constant. (a) Determine the value of v that minimizes E. (b) Sketch the graph of E.

Solution. (a) Note that a, L, and u are constants greater than 0 and that the domain of $E(v)$ is $v > u$. One can compute the derivative of E using the quotient rule:

$$\begin{aligned} E'(v) &= aL \left(\frac{v^3}{v-u} \right)' \\ &= aL \left(\frac{3v^2(v-u) - v^3}{(v-u)^2} \right) \\ &= aLv^2 \left(\frac{2v-3u}{(v-u)^2} \right) \end{aligned}$$

As $v = 0$ is not in the domain, we see that $E'(v) = 0$ only when $2v - 3u = 0$, i.e. $v = \frac{3}{2}u$. When $u < v < \frac{3}{2}u$, $E'(v) < 0$, and when $v > \frac{3}{2}u$, $E'(v) > 0$. Thus, by the First Derivative Test for Absolute Extreme Values, $v = \frac{3}{2}u$ gives the absolute minimum for E. (b) To sketch the graph of E we can follow the preparatory steps outlined in section 4.5. We have already noted that the domain of E is $v > u$. The function E has no x- or y-intercepts. (The point (0,0) would be an intercept, but it is outside the domain.) There is no apparent symmetry. The function E is defined everywhere in its domain, so there can be no vertical asymptotes except as v approaches u from the right. As $v \to u^+$, the numerator of E, aLv^3, approaches aLu^3, while the denominator of E, $v - u$, approaches 0 but always remains positive. Thus, $\lim_{v \to u^+} E(v) = \infty$. Now let's consider the behavior of E as v approaches ∞. Note that

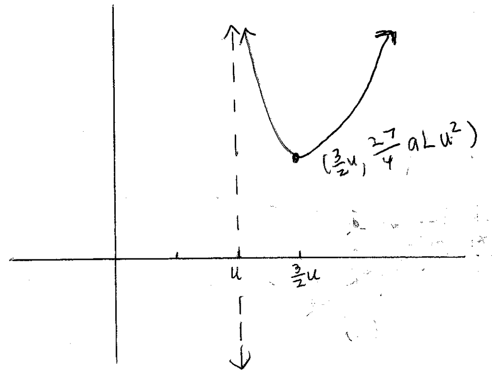
$$\lim_{v \to \infty} E(v) = \lim_{v \to \infty} \frac{aLv^3}{v-u} = \lim_{v \to \infty} \frac{aLv^2}{1 - \frac{u}{v}}$$

The numerator of the last limit displayed approaches ∞, while the denominator approaches 1. Thus the limit in general is ∞. Moreover, this limit evidently does not yield a slant aymptote. As was deduced in part (a), E decreases on $(u, \frac{3}{2}u)$ and increases on $(\frac{3}{2}u, \infty)$. Its only local minimum is its absolute minimum $(\frac{3}{2}u, E(\frac{3}{2}u)) = (\frac{3}{2}u, \frac{27}{4}aLu^2)$. To find the second derivative of E one can use the quotient rule again. After simplification one obtains

$$E''(v) = 2aLv \left(\frac{v^2 - 3uv + 3u^2}{(v-u)^3} \right)$$

$2aLv$ and $(v-u)^3$ are clearly positive, but I claim that $v^2 - 3uv + 3u^2$ is too. To see this one may complete the square to obtain $(v - \frac{3}{2}u)^2 - \frac{9}{4}u^2 + 3u^2$, which is obviously positive. Thus, E is always concave up. Putting all of these observations together, one obtains the following sketch:

§ 4.7 #42(b)

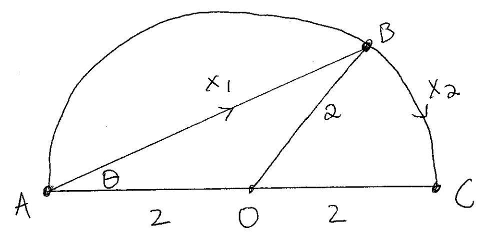


$\square$

§4.7 # 46. A woman at a point A on the shore of a circular lake with radius 2 mi wants to arrive at the point C diametrically opposite A on the other side of the lake in the shortest possible time. She can walk at the rate of 4 mi/h and row a boat at 2 mi/h. How should she proceed?

Solution. We assume the woman must row none, some, or all of the way and then walk the rest. Let θ, x_1, and x_2 be defined as suggested by the diagram below. The time required to travel this path is $T = \frac{x_1}{2} + \frac{x_2}{4}$. We can express x_2 in terms of θ by recalling the fact that the length of an arc of an inscribed angle θ is $2r\theta$ where r is the radius of the circle. Thus, $x_2 = 4\theta$ in this case. Moreover, we can express x_1 in terms of θ by using the law of cosines. We get $2^2 = x_1^2 + 2^2 - 2(2)(x_1)\cos\theta$, and so $4x_1 \cos\theta = x_1^2$. Certainly if $x_1 \neq 0$ then it follows that $4\cos\theta = x_1$, but it turns out that this equation also holds when $x_1 = 0$. To see this, note that when $x_1 = 0$, $\theta = \frac{\pi}{2}$, so $4\cos\theta = 0$ as well. Thus, we have obtained the following formula for the time required: $T(\theta) = \frac{4\cos\theta}{2} + \frac{4\theta}{4} = 2\cos\theta + \theta$, where $\theta \in [0, \frac{\pi}{2}]$. Differentiating yields $T'(\theta) = -2\sin\theta + 1$. It follows that $T'(\theta) = 0$ only when $\theta = \frac{\pi}{6}$. We are interested in finding the absolute minimum of T, and it suffices to consider only the three values $T(0)$, $T(\frac{\pi}{2})$, and $T(\frac{\pi}{6})$, according to the Closed Interval Method (p. 275). Since $T(0) = 2\cos 0 + 0 = 2$, $T(\frac{\pi}{2}) = 2\cos\frac{\pi}{2} + \frac{\pi}{2} = \frac{\pi}{2} < 2$, and $T(\frac{\pi}{6}) = 2\cos\frac{\pi}{6} + \frac{\pi}{6} = \sqrt{3} + \frac{\pi}{6} > 2$, we see that $\theta = \frac{\pi}{2}$ gives the absolute minimum. That is, the woman should not use the rowboat at all and only walk around the edge of the lake to reach point C in the fastest possible way.

§4.7 #46



□

§4.7 # 53(a). If $C(x)$ is the cost of producing x units of a commodity, then the **average cost** per unit is $c(x) = C(x)/x$. Show that if the average cost is minimum, then the marginal cost equals the average cost.

Solution. If the average cost is a local minimum, then the derivative of the average cost is zero. Moreover, using the quotient rule, we can obtain

$$c'(x) = \left(\frac{C(x)}{x}\right)' = \frac{C'(x)x - C(x)}{x^2}$$

Thus, if $c'(x) = 0$, it follows that $C'(x)x = C(x)$, or in other words $C'(x) = c(x)$. □

§4.7 # 72. The blood vascular system consists of blood vessels (arteries, arterioles, capillaries, and veins) that convey blood from the heart to the organs and back to the heart. This system should work so as to minimize the energy expended by the heart in pumping the blood. In particular, this energy is reduced when the resistance of the blood is lowered. One of Poiseuille's Laws gives the resistance R of the blood as

$$R = C\frac{L}{r^4}$$

where L is the length of the blood vessel, r is the radius, and C is a positive constant determined by the viscosity of the blood. The figure shows a main blood vessel with radius r_1 branching at an angle θ into a smaller vessel with radius r_2. (a) Use Poiseuille's Law to show that the total resistance of the blood along the path ABC is

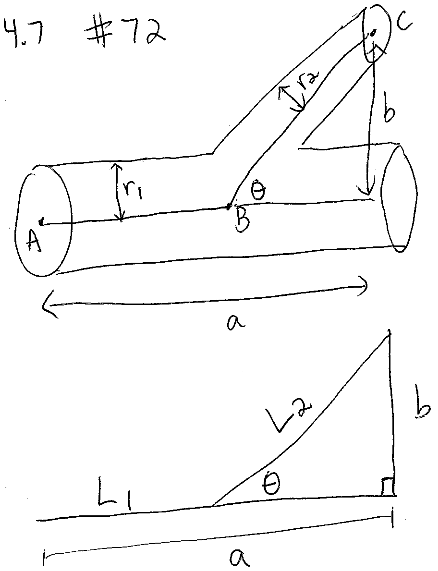
$$R = C\left(\frac{a - b\cot\theta}{r_1^4} + \frac{b\csc\theta}{r_2^4}\right)$$

where a and b are the distances shown in the figure. (b) Prove that this resistance is minimized when

$$\cos\theta = \frac{r_2^4}{r_1^4}$$

(c) Find the optimal branching angle (correct to the nearest degree) when the radius of the smaller blood vessel is two-thirds the radius of the larger vessel.

§4.7 #72



Solution. (a) We may divide the path ABC into two paths, one from A to B and one from B to C. We can apply Poiseuille's Law to each path separately and then add up the resistances. Let L_1 be the length from A to B and let L_2 be the length from B to C. From the simplified diagram, we see that $\csc\theta = \frac{b}{L_2}$, so $L_2 = b\csc\theta$. Similarly, $\cot\theta = \frac{b}{a-L_1}$, so $L_1 = a - b\cot\theta$. Now, using Poiseuille's Law for the paths AB and BC, we get

$$\begin{aligned} R &= C\frac{L_1}{r_1^4} + C\frac{L_2}{r_2^4} \\ &= C\frac{a - b\cot\theta}{r_1^4} + C\frac{b\csc\theta}{r_2^4} \end{aligned}$$

as desired. (b) Recall we are assuming that r_1 and r_2 are fixed positive constants such that $r_1 > r_2$. We wish to find the absolute minimum to the function

$$R(\theta) = C\left(\frac{a - b\cot\theta}{r_1^4} + \frac{b\csc\theta}{r_2^4}\right)$$

where the domain of R is $\theta \in (0, \pi)$. The derivative of R is calculated as follows:

$$\begin{aligned} R'(\theta) &= C\left(\frac{-b}{r_1^4}(-\csc^2\theta) + \frac{b}{r_2^4}(-\csc\theta\cot\theta)\right) \\ &= \frac{Cb}{\sin^2\theta}\left(\frac{1}{r_1^4} - \frac{\cos\theta}{r_2^4}\right) \end{aligned}$$

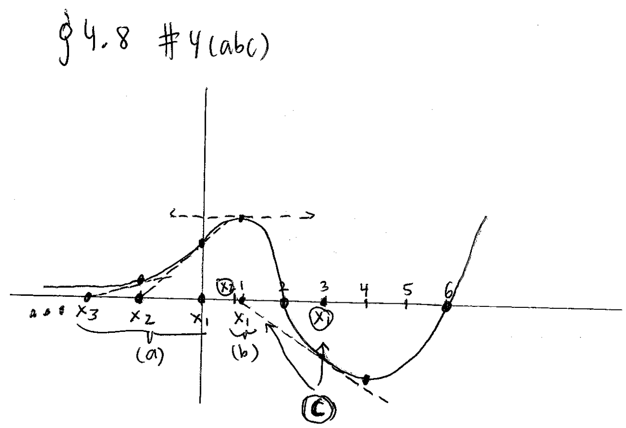
Since $r_1 > r_2 > 0$, it follows that $0 < \frac{r_2^4}{r_1^4} < 1$, so there is some $\theta_0 \in (0, \pi)$ such that $\cos\theta_0 = \frac{r_2^4}{r_1^4}$. Note that $R'(\theta_0) = 0$. Furthermore, if $0 < \theta < \theta_0$, then since cos is decreasing on $(0, \pi)$ it follows that $R'(\theta) < 0$. Similarly, if $\theta_0 < \theta < \pi$, then $R'(\theta) > 0$. Thus, θ_0 is the absolute minimum for R by the First Derivative Test for Absolute Extreme Values. (c) We are given $r_2 = \frac{2}{3}r_1$ and we may compute

θ using the result from part (b). We get

$$\cos\theta = \frac{(\frac{2}{3}r_1)^4}{r_1^4} = \frac{16}{81}$$

Thus, $\theta = \cos^{-1}\left(\frac{16}{81}\right) \approx 78.6°$. □

§4.8 # 4(abc). For each initial approximation, determine graphically what happens if Newton's method is used for the function whose graph is shown. (a) $x_1 = 0$ (b) $x_1 = 1$ (c) $x_1 = 3$



Solution. (a) If $x_1 = 0$, then we see from the graph and relevant tangent lines that x_n gets farther and farther to the left. (b) If $x_1 = 1$, then the first tangent line is horizontal and so we can't find x_2. (c) If $x_1 = 3$, it seems that the tangent line will cross the x-axis to the left of $x = 1$. In this case we're back to a situation as in part (a). Of course, if the graph were drawn more precisely and it became apparent that this tangent line actually crossed exactly at $x = 1$ or slightly to the right of $x = 1$, then the behavior of the sequence x_n would naturally be quite different. □

§4.8 # 29(a). Apply Newton's method to the equation $x^2 - a = 0$ to derive the following square-root algorithm (used by the ancient Babylonians to compute $\sqrt{a}$):

$$x_{n+1} = \frac{1}{2}\left(x_n - \frac{a}{x_n}\right)$$

Solution. We seek an approximation for the positive root of the function $f(x) = x^2 - a$. Since $f'(x) = 2x$, applying Newton's method gives the sequence $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - a}{2x_n} = \frac{1}{2}\left(x_n - \frac{a}{x_n}\right)$, as desired. □

§4.8 # 30(a). Apply Newton's method to the equation $1/x - a = 0$ to derive the following reciprocal algorithm:

$$x_{n+1} = 2x_n - ax_n^2$$

Solution. Let $f(x) = 1/x - a$. Then $f'(x) = -1/x^2$. So Newton's method applied to this function gives the sequence $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{1/x_n - a}{-1/x_n^2} = x_n - \left(ax_n^2 - x_n\right) = 2x_n - ax_n^2$, as desired. □

§4.8 # 38. Of the infinitely many lines that are tangent to the curve $y = -\sin x$ and pass through the origin, there is one that has the largest slope. Use Newton's method to find the slope of that line correct to six decimal places

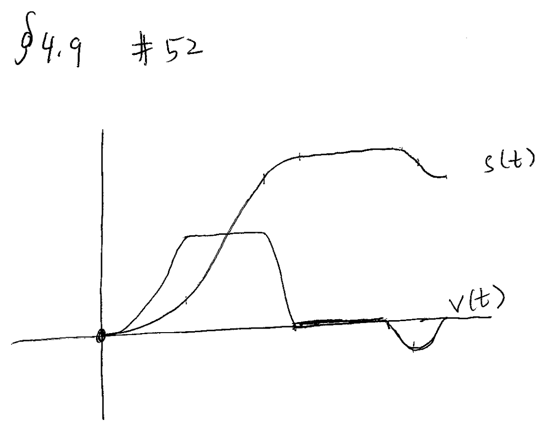
Solution. Given a point $(x_0, -\sin x_0)$ on the graph of $-\sin x$, the equation for the tangent line is $y + \sin x_0 = -\cos x_0(x - x_0)$. This line goes through the origin whenever $\sin x_0 = -\cos x_0(-x_0)$, or $\tan x_0 - x_0 = 0$. From the graph of $-\sin x$ it's clear that such a tangent line with a maximum possible slope will occur somewhere in the interval $x_0 \in (\pi, 3\pi/2)$. Also, since there's only one root of $\tan x - x$ in this interval, it is reasonable to expect that applying Newton's method will work. However, to make sure that the first linear approximation's root remains inside the interval, let's choose a starting point $x_1 = 4.5$ slightly less than $3\pi/2$ so that the linear approximation is nearly vertical. As the derivative of $\tan x - x$ is $\sec^2 x - 1$, the formula we should use is

$$x_{n+1} = x_n - \frac{\tan x_n - x_n}{\sec^2 x_n - 1}$$

Using a calculator we get: $x_2 \approx 4.49361390$, $x_3 \approx 4.49340966$, $x_4 \approx 4.49340946$, and $x_5 \approx 4.49340946$. We see that the root correct to 6 decimal places is 4.493409. $\square$

§4.9 # 52 The graph of the velocity function of a particle is shown in the figure. Sketch the graph of the position function. Assume $s(0) = 0$.

Solution. A sketch is given below. Note in particular that in the region where the velocity function $v(t)$ is constant and positive, your position graph should be a straight line with positive slope.



$\square$

§4.9 # 64. Show that for motion in a straight line with constant acceleration a, initial velocity v_0, and initial displacement s_0, the displacement after time t is

$$s = \frac{1}{2}at^2 + v_0 t + s_0$$

Solution. If a is constant then v, the antiderivative of a, is $at + v_0$. Then the displacement s is the antiderivative of $v = at + v_0$, or $\frac{1}{2}at^2 + v_0 t + s_0$. $\square$

§4.9 # 70. The linear density of a rod of length 1 m is given by $\rho(x) = 1/\sqrt{x}$, in grams per centimeter, where x is measured in centimeters from one end of the rod. Find the mass of the rod.

Solution. As in an Example 3.7.2 on page 223, we are meant to assume that linear density is the derivative of mass. More precisely, let $m(x)$ be the function that tells you the mass in grams of the $[0, x]$ portion of the rod where x is measured in cm. Then $m(0) = 0$ and m is the antiderivative of the linear density $\rho(x) = 1/\sqrt{x}$. The antiderivative of $1/\sqrt{x} = x^{-1/2}$ is $2x^{1/2} = 2\sqrt{x}$. The mass of the whole rod is then simply $m(100) = 2\sqrt{100} = 20$ grams. $\square$