Math 274: Topics in Algebra

Arthur Ogus

Text: Under development
Prerequisites: Math 250, Math 256A

Logarithmic geometry was originated in the eighties by Deligne, Faltings, Illusie, Fontaine and Kato. It provides a powerful conceptual framework for dealing with two related and fundamental problems in algebraic geometry: compactification and degeneration. The classical techniques for dealing with these problems include toroidal embeddings, the theory of semistable reduction, and differential equations with regular singularities. Logarithmic geometry puts all these into a common unified geometric setting. As Kato has said, a logarithmic structure is a "magic powder" which can be added to a "bad" situation to transform it into a "good" one. The first important applications of log geometry were to arithmetic and p-adic Hodge theory, but more recently it has also been applied to many geometric problems, especially mirror symmetry. The basic notion of a logarithmic scheme is appealingly simple, but working out the foundations thoroughly does pose some serious challenges which I will try to address.

Although this material is fairly new, the course will be foundational and hence elementary, and I will try to make it accessible to students who have completed Math 256A. After a brief motivational introduction, we will cover the foundations of toric geometry (the geometry of commutative monoids) which form the technical underpinnings of log geometry. Although much of the material should be familiar to experts in toric geometry, I expect that the study of homomorphisms of monoids, which play an important role in log geometry, may be new. After sheafifying these concepts, we will get to log structures, log schemes, log smoothness, and log differentials. As time permits I will discuss topological invariants of log schemes, especially their singular and de Rham cohomology. I also hope that experts will help us explore the relation between log and tropical geometry.

The first part of the course, devoted to monoids and cones and homomorphisms between them, will be quite elementary. Later we will need some basic concepts from algebraic geometry: sheaves, schemes, and differentials. I will be distributing notes for my book during the course which will serve as a "text." The required assignments will be to find non trivial mistakes

and/or fill in some missing proofs.