PDE LEARNING SEMINAR TOPICS - FALL 2024

The study of the stability of solitons in nonlinear dispersive and hyperbolic equations is a rich and vast subject. This semester's learning seminar is on soliton dynamics. There are some nice survey articles like [MS04], [Sof06], [LM14] and [KMM16].

1. Existence, uniqueness and symmetry property of ground states :
   - [Tao06, Appendix B] *Construction of ground states*
   - [GNN79] *Symmetry and related properties via the maximum principle*
   - [CGS89] *Asymptotic symmetry and local behavior of semilinear elliptic equations with critical Sobolev growth*
2. Orbital stability via variational approach :
   - [CL82] *Orbital stability of standing waves for some nonlinear Schrödinger equations*
   - [GSS87] *Stability theory of solitary waves in the presence of symmetry. I*
3. Normal form transformation and vector field method for Klein-Gordon :
   - [Sha85] *Normal forms and quadratic nonlinear Klein-Gordon equations*
   - [Kla87] *Global existence of small amplitude solutions to nonlinear Klein-Gordon equations in four spacetime dimensions*
4. Classical modulation theory :
   - [Wei86] *Lyapunov stability of ground states of nonlinear dispersive evolution equations*
   - [Wei85] *Modulational stability of ground states of nonlinear Schrödinger equations*
   - [Stu01] *Modulational approach to stability of non-topological solitons in semilinear wave equations*
5. Asymptotic stability of KdV :
   - [PW94] *Asymptotic Stability of Solitary Waves*
   - [MM01], [MM02], [MM05] *Asymptotic stability of solitons for subcritical generalized KdV equations*
6. Kink solutions - Strichartz or virial estimates :
   - [KMM17] *Kink dynamics in the ϕ^4 model: asymptotic stability for odd perturbations in the energy space*
   - [Kow+21] *A sufficient condition for asymptotic stability of kinks in general $(1+1)$-scalar field models*
   - [KM22] *Kink dynamics under odd perturbations for (1+1)-scalar field models with one internal mode*
7. Kink solutions - distorted Fourier transformation :
   - [LS23] *Asymptotic stability of the sine-Gordon kink under odd perturbations*
   - [LS24] *On codimension one stability of the soliton for the 1D focusing cubic Klein-Gordon equation*
   - [GP22] *Quadratic Klein-Gordon equations with a potential in one dimension*

This is a tentative list of topics for the nonlinear PDE learning seminar at UC Berkeley during Fall 2024.

8. Some topological solitons :
   - [GNT10] *Asymptotic stability, concentration, and oscillation in harmonic map heat-flow, Landau-Lifshitz, and Schrödinger maps on* $\mathbb{R}^2$
9. Geodesic hypothesis :
   - [Stu04] *The geodesic hypothesis and non-topological solitons on pseudo-Riemannian manifolds*
   - [Yan14] *On the geodesic hypothesis in general relativity*

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