TOPICS FOR SECOND MIDTERM, MATH 55 SPRING 2016

You must know all definitions and proofs presented in class, as well as proofs that I told you to read in the book.

Chapters 1 and 2. Everything in these chapters is a prerequisite. All proofs must be written in complete English sentences (sentences which have verbs and end with periods) with appropriate use of quantifiers. The definitions of sets, functions, bijections, sequences, and cardinality in Chapter 2 are particularly important.

Chapter 4. I will not ask you any questions about modular arithmetic, but you are expected to be comfortable with the basic definitions of divisibility, prime, gcd, and Euclid's lemma. I may ask you to prove things about integers using induction or pigeonhole, as well as probability questions about divisibility. In any case this will be a big part of the final exam so it is worth reviewing.

Chapter 5. Main topics:

- Induction
- Strong induction
- Well-ordering principle
- Recursive definitions

Types of problems:

- Proofs using induction and its variants, including proofs of statements about topics in other chapters.

Chapter 6. Main topics

- Cardinality, product and sum rules
- Bijections, many to one maps, division rule
- Permutations and combinations
- Combinatorial proofs
- Pigeonhole Principle (including Generalized Pigeonhole Principle)
- Binomial coefficients, Binomial Theorem, Pascals identity

Types of problems

- Basic counting using product and sum rule (by defining a process for constructing the objects you want to count)
- Proofs using the pigeonhole principle
- Counting permutations and combinations with and without repeated objects
- Distributing indistinguishable objects into distinguishable boxes.
- Proofs involving binomial coefficients, including combinatorial proofs (i.e., counting the same thing in two different ways)

Chapter 7. Main topics

- Probability (experiment, sample space, event, outcome, probability distribution)
- Conditional probability, independent events, law of total probability
- Bayes Theorem
- Coin flips (Bernoulli Trials), the binomial distribution, the geometric distribution
- Random variables, expected value, linearity of expectation
- Independent random variables
- Variance and Chebyshev's inequality

Types of problems

- Computing probabilities and conditional probabilities (using basic definitions and counting)
- Applications of Bayes Theorem
- Determining whether events and random variables are independent
- Computing expectation and variance of random variables
- Applying linearity of expectation
- Applying Chebyshev's inequality
- Proofs of basic properties of expected value and variance