Math 55 Spring 2016 Practice Midterm 1

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80 minutes, closed book, closed notes

1. (3 pts each) True or False (and provide a brief one or two sentence explanation):

   (a) The compound propositions

   $$\neg p \to q$$

   and

   $$\neg p \vee \neg q$$

   are logically equivalent.

   (b) The compound proposition

   $$(p \to F) \vee (p \to T)$$

   is a tautology, where T and F are true and false.

   (c) Every subset of the integers has a least element.

   (d) If A and B are uncountable then $A \cup B$ is also uncountable.

2. (7 pts) Suppose A, B, and C are sets such that $A \cap C = B \cap C$ and $A \cup C = B \cup C$. Can you conclude that $A = B$? Give a proof or a counterexample.

3. (7 pts) Suppose A, B, and C are sets and $f : A \to B$ and $g : B \to C$ are functions such that $g \circ f : A \to C$ is injective. Can you conclude that both f and g are injective? Give a proof or a counterexample.

4. (7 pts) Prove that if x and y are integers and p is a prime such that xy and $x + y$ are both divisible by p, then both x and y must be divisible by p.

5. (7 pts) Prove that if n is an integer then $n^2 \equiv 0$ or $1 (\text{mod } 4)$. Use this to show that if $m = 4k + 3$ for some integer k then m cannot be written as the sum of the squares of two integers.

6. (5 pts each) (a) Find an inverse of 5 modulo 13. (b) Compute the remainder when 3^{16} is divided by 11.