1.1, 7 Let us organize the information. Immediately we can check that (a) is false, (b) is true, and (c) is true. For (d), recall that “if $P$ then $Q$” is satisfied if $P$ is false, or if $P$ is true and $Q$ is true. The condition $P$ is not satisfied, so the statement is automatically true (“Quixote Media” had the lowest net profit” is false). For (e), recall that “$P$ if and only if $Q$” is true when $P$ and $Q$ are both true or both false. Both statements are true, so it evaluates to true.

1.1, 9bdf (b) Swimming at the New Jersey shore is allowed and sharks have been spotted near the shore. (d) If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore. (f) If swimming at the New Jersey shore is not allowed, the sharks have not been spotted near the shore.

1.1, 12bdf (b) You pass the course exactly when you do not miss the final examination. (d) You have the flu or you miss the final examination or you pass the course. (f) You have the flu and miss the final examination, or you do not miss the final examination and pass the course.

1.1, 18 (a) true (the condition is not satisfied). (b) true (same reason as a). (c) false (the condition $1 + 1 = 2$ is true, but dogs cannot fly). (d) true (both $2 + 2 = 4$ and $1 + 2 = 3$ are true).

1.1, 23 (a) If the wind blows from the northeast, then it snows. (b) If it stays warm for a week, then the apple trees will bloom. (c) If the Pistons win the championship, then they beat the Lakers. (d) If you get to the top of Long’s Peak, then you walk 8 miles. (e) If you are world-famous, you get tenure as a professor. (f) If you drive more than 400 miles, then you will need to buy more gasoline. (g) If your guarantee is good, then you bought your CD player less than 90 days ago. (h) If the water is not too cold, then Jan will go swimming.

1.1, 27 (a) Converse: If I ski tomorrow, then it snowed today. Contrapositive: If I do not ski tomorrow, then it did not snow today. Inverse: If it does not snow today, then I will not ski tomorrow. (b) Converse: If I come to class, then there is going to be a quiz. Contrapositive: If I do not come to class, then there is no quiz. Inverse: If I do not come to class, then there is not going to be a quiz. (c) Converse: If a positive integer has no prime divisors other than 1 and itself, then it is a prime. Contrapositive: If a positive integer has a prime divisor other than 1 and itself, then it is not a prime. Inverse: If a positive integer is not a prime, then it has a prime divisor other than 1 and itself.

1.2, 5 $e \rightarrow a \land (b \lor (p \land r))$

1.2, 16 (a) If the cannibal is a liar, he will say no. If he is not a liar, he will also say no. The explorer gets no information. (b) Any question that is true or false without conditions will do. For example, “Is $1 + 1 = 2$?”

1.2, 18 Let $J$ mean Jasmine attends, $S$ mean Samir attends, and $K$ mean Kanti attends. We have the implications

$J \rightarrow \neg S$
Samir can never attend this party. If he does, then Kanti does. If Kanti does, then Jasmine does. But if Jasmine does, then Samir does not, a contradiction.

If Kanti attends, then Jasmine does as well. So the possibilities are: no one attends, Jasmine attends, and Kanti and Jasmine attend.

1.3, 7 (a) Jan is neither happy nor rich. (b) Carlos will not bike and he will not run tomorrow. (c) Mei does not walk and she does not take the bus to class. (d) Ibrahim is not smart nor is he hard working.

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1.3, 9 (a)
1.3, 11 (a) \[ p \land q \rightarrow p \equiv \neg(p \land q) \lor q \equiv (\neg p \lor \neg q) \lor q \equiv \neg p \lor (\neg q \lor q) \equiv p \lor T \equiv T \]

(b) \[ p \rightarrow p \lor q \equiv \neg p \lor (p \lor q) \equiv (\neg p \lor p) \lor q \equiv T \lor q \equiv T \]

(c) \[ \neg p \rightarrow (p \rightarrow q) \equiv p \lor (p \rightarrow q) \equiv p \lor (\neg p \lor q) \equiv (p \lor \neg p) \lor q \equiv T \lor q \equiv T \]

(d) \[ (p \land q) \rightarrow (p \rightarrow q) \equiv (p \land q) \rightarrow (\neg p \lor q) \equiv \neg(p \land q) \lor (\neg p \lor q) \equiv \neg p \lor p \lor (\neg q \lor q) \equiv \neg p \lor T \equiv T \]

(e) \[ \neg(p \rightarrow q) \rightarrow p \equiv \neg \neg(p \rightarrow q) \lor p \equiv (\neg p \lor q) \lor p \equiv (\neg p \lor p) \lor q \equiv T \lor q \equiv T \]

(f) \[ \neg(p \rightarrow q) \rightarrow \neg q \equiv \neg \neg(p \rightarrow q) \lor \neg q \equiv (p \land \neg q) \lor \neg q \equiv \neg(p \land \neg q) \lor \neg q \equiv \neg p \lor q \lor \neg q \equiv T \]

1.3, 21 The proposition \( \neg(p \leftrightarrow q) \) is false exactly when \( p \) and \( q \) have the same truth values. The proposition \( \neg p \leftrightarrow q \) is true exactly when \( p \) and \( q \) have different truth values.

1.4, 7 (a) All comedians are funny. (b) All people are funny and comedians. (c) There is a comedian who is funny. (d) There is a person who is a comedian and funny. Note the difference between (c) and (d); (c) can be satisfied if there simply aren’t any comedians.

1.4, 9 (a) \( \exists x P(x) \land Q(x) \) (b) \( \exists x P(x) \land \neg Q(x) \) (c) \( \forall x P(x) \lor Q(x) \) (d) \( \forall x \neg P(x) \land \neg Q(x) \)

1.4, 15 (a) true, squares of integers are nonnegative (b) false; the integers do not have a number which squares to 2 (c) true; check positive and negative cases separately (d) false; this is opposed to (a)

1.4, 19 (a) \( P(1) \lor P(2) \lor P(3) \lor P(4) \lor P(5) \)
(b) \( P(1) \land P(2) \land P(3) \land P(4) \land P(5) \)
(c) \( P(1) \land P(2) \land P(3) \land P(4) \land P(5) \)
(d) \( P(1) \lor P(2) \lor P(3) \lor P(4) \lor P(5) \)
(e) \( P(1) \land P(2) \land P(4) \land P(5)) \lor (\neg P(1) \lor \neg P(2) \lor \neg P(3) \lor \neg P(4) \lor \neg P(5)) \)

1.4, 43 They are not logically equivalent. For example, take \( P(x) := x \) is odd and \( Q(x) := 1 = 2 \). Then, \( \forall x(P(x) \rightarrow Q(x)) \) is false; there are some \( x \) which are odd but for which \( 1 = 2 \) is false. However, \( \forall xP(x) \rightarrow \forall xQ(x) \) is true, since \( \forall xP(x) \) is false and so the implication is true.

1.5, 9ace (a) \( \forall x L(x, Jerry) \)
(c) \( \exists x \forall y L(y, x) \)
(e) \( \exists x \neg L(Lydia, x) \)
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1.5, 20 (a) \( \forall x \forall y(x < 0) \land (y < 0) \rightarrow (xy > 0) \)
(b) \( \forall x \forall y(x < 0) \land (y < 0) \rightarrow (\frac{x+y}{2} > 0) \) (this one is a bit weird to me because the average might not be an integer, so it seems kind of not well-defined actually)
(c) \( \exists x \exists y(x < 0) \land (y < 0) \land (x - y \geq 0) \)
(d) \( \forall x \forall y |x + y| \leq |x| + |y| \)

1.5, 25 (a) There is an \( x \) such that for all \( y \), \( xy = y \).
(b) For all negative \( x \) and negative \( y \), \( xy \) is negative.
(c) There is an \( x \) and \( y \) such that \( x^2 > y \) and \( x < y \).
(d) For all \( x \) and \( y \), there is a \( z \) such that \( x + y = z \).

1.5, 30 (a) \( \forall y \forall x \neg P(x, y) \)
(b) \( \exists x \forall y \neg P(x, y) \)
(c) \( \forall y (\neg Q(y) \lor \exists x R(x, y)) \)
(d) \( \forall y (\forall x \neg R(x, y) \land \exists x \neg S(x, y)) \)
(e) \( \forall y (\exists x \forall z \neg T(x, y, z) \land \forall x \exists z \neg U(x, y, z)) \)

1.6, 2 It is valid.

1.6, 3bd (b) simplification (d) modus tollens

1.6, 6 Let \( r \) mean it rains, \( f \) mean it is foggy, \( s \) mean the sailing race is held, \( \ell \) mean the lifesaving demo goes on, and \( t \) mean the trophy is awarded. We have

\[-r \lor \neg f \rightarrow s \land \ell\]

\[s \rightarrow t\]

\[\neg t\]

Our reasoning is as follows

| \( \neg t \) | given |
| \( s \rightarrow t \) | given |
| \( \neg s \) | modus tollens on the above two |
| \( \neg s \lor \neg \ell \) | addition |
| \( \neg (s \land \ell) \) | logical equivalence |
| \( \neg r \lor \neg f \rightarrow (s \land \ell) \) | given |
| \( \neg (\neg r \lor \neg f) \) | modus tollens |
| \( r \land f \) | negation |
| \( r \) | simplification |

1.6, 7 Modus pollens

1.6, 15 (a) Correct. Modus pollens. (b) Incorrect. Affirms the conclusion. (c) Incorrect. Denying the hypothesis. (d) Correct. Modus tollens.

1.6, 19 (a) Incorrect. Affirms the conclusion. (b) Correct. Modus tollens. (c) Incorrect. Denies the hypothesis.