The Midterm will cover all assigned sections from Chapters 1, 2, and 3 (lectures 1-8). You should know all the definitions and theorems we have covered in lecture. Knowing a definition means you should be able to state it precisely, and you should know how to go about checking whether a given object satisfies the definition or not. You should be aware of what *type* of object (scalar, vector, matrix, linear system, linear transformation) is appropriate whenever you use any of these terms.

Key definitions:

- linear system, existence, uniqueness,consistency, solution space.
- augmented matrix, row operations, row reduction, pivots, REF, RREF.
- pivot variables and free variables, general solution.
- vector, $\mathbb{R}^n$, linear combination, span.
- matrix vector product and relation to linear combinations.
- homogeneous and inhomogeneous systems.
- linear dependence and independence.
- linear transformation, 1-1 and onto function.
- matrix transformation, standard matrix of a linear transformation.
- matrix multiplication, composition of linear transformations.
- inverse, invertibility in terms of 1-1 and onto, linear independence and span, and pivots.
- linear subspace, column space and null space of a matrix.
- dimension of a subspace, rank of a matrix.
- coordinates with respect to a basis.
- determinant, properties with respect to row operations and multiplication.

I can't stress how important it is to know the definitions. In many conceptual problems, the best way to start is: write down the definitions of everything that is given, and then write down the definitions relevant to whatever the problem is asking you to show. Then ask: what theorems do I know that connect these definitions?

Key Theorems: The theorems we have covered describe how the definitions above are related to each other. The beauty and power of linear algebra comes from the fact that there are lots and lots of connections and they fit together very nicely. You should know the statements all the theorems stated in the lecture notes and in the assigned sections of the textbook (if I stated some theorem slightly differently than the book, either version is acceptable).

Some of the theorems just state basic properties of the definitions, which are crucial to know. Some of them are deeper and more surprising, and it is important to understand their proofs — without understanding the proofs, you will not understand *why* the connections work, and how to use them. The most important theorems in this regard are: Ch1: Theorems 4*, 7*, 8*, 10, 11*, 12*; Ch2: Theorems 5, 7, 8*, 9, 12, 13, 14. Ch3: Theorem 3, 4, 6.

On the midterm, I may ask you to prove one the theorems above that are starred (I won't ask about the other ones). The proof does not have to be entirely formal or identical to the one in the book or my lecture, but I want a clear explanation of why the theorem is true.

Algorithms. There is only one algorithm so far in this course: Row Reduction. This algorithm is used both to solve concrete problems (such as solving a linear system, finding a basis of the null space of a matrix, or computing the determinant), as well as to prove theorems (by reasoning about pivots). You should also know how to compute a determinant using the recursive formula defined in terms of submatrices.

Types of Problems. The problems will be similar to the homework problems, but to help you study, here are the main kinds:

- Given a linear system: determine whether it is consistent, find its general solution, reduce it to REF or RREF, determine whether it is consistent for all right hand sides.
- Given a set of vectors: determine their span, whether they are linearly independent, whether another vector is in the span. Find a basis for the span.
- Given a matrix: determine its inverse, multiply it by another matrix, determine whether its columns are linearly independent/span. Find bases for its column space and null space. Determine the dimension of its column space and null space.
- Given a linear transformation: write its standard matrix, determine whether it is 1-1/onto, find its inverse or composition with another linear transformation.
- Determine whether a given transformation is linear, whether a given set is a linear subspace.
- Compute the determinant of a matrix.
- Give an example of a matrix/vector/transformation with a specified property.
- Variations on and combinations of all of the above. I will often give you a problem that requires you to solve several of the above as subproblems.
- State a theorem from class and give a proof of it.
- True/False questions.

I will most likely give you problems that involve combining ideas from multiple chapters.

> **SHOW YOUR WORK. If you just write a numerical answer without a clear description of the process you used to get it, you will only get partial credit. If something really is obvious (such as saying that the vector $(1, 1)$ is orthogonal to $(1, -1)$) use the words "observe that" or "by inspection" to indicate this.**