The Midterm will cover all assigned sections from Chapters 4, 3, 5, and 6 (up to 6.5), as well as chapters 1 and 2 (in any case, it is impossible to think about the later chapters without those). You should know all the definitions and theorems we have covered in lecture. Knowing a definition means you should be able to state it precisely, and you should know how to go about checking whether a given object satisfies the definition or not.

Key definitions and concepts:

- Rank of a matrix (see lectures 7 and 10 and 4.6).
- Vector space, subspace, linear combination, linear (in)dependence, span, basis, linear transformation, 1-1 and onto, kernel, image (in the abstract setting). Dimension.
- Coordinate vector, coordinate mapping, correspondence principle, isomorphism.
- Matrix of a linear transformation with respect to input and output bases.
- Change of basis, change of coordinates matrix.
- Determinant.
- Eigenvalue, eigenvector, eigenspace. Characteristic polynomial.
- Diagonalizable, similar.
- Dot product, norm, distance, orthogonal set, orthogonal basis, orthonormal set/basis.
- Orthogonal projection, orthogonal complement.
- Least squares solution, normal equations.
- Transpose.

Key Theorems: (Chapter.Theorem)

You need to know the statements of and understand how to use the following theorems. Almost all of these are contained in the lecture notes. You may be asked to give the proofs of the starred ones. The proof does not have to be entirely formal or identical to the one in the book, but I want a clear explanation of why the theorem is true.

Note that there are very important concepts (such as how to find the matrix of a linear transformation) which are not on this list because they are not explicitly encapsulated as theorems.

- Rank nullity theorem (4.14).
- * Every vector in a vector space is a unique linear combination of vectors in a basis (4.7).
- * A set of vectors is independent iff no vector is in the span of the previous ones (4.4).
- * The span of a set of vectors is always a subspace (4.1). The Kernel and Image of a linear transformation are subspaces.
- * If V has a basis with n vectors, then every linearly independent set has $\leq n$ vectors and every spanning set has $\geq n$ vectors (4.9, 4.10).
- Every linearly independent set of $< n$ vectors may be extended to a basis, and every spanning set of $> n$ vectors may be trimmed to a basis (4.5, 4.11).
- The coordinate mapping is an isomorphism (correspondence principle).
- * Change of basis theorem (4.15).
- Invertible matrix theorem.
- Properties of Determinants (3.1, 5.3).
- * The eigenvalues of a matrix are the roots of its characteristic polynomial. The eigenvalues of a triangular matrix are its diagonal entries (5.1).
- * A matrix is diagonalizable iff it there is a basis of linearly independent eigenvectors (5.5).
- Eigenvectors from different eigenspaces are linearly independent (5.2).
- If an $n \times n$ matrix has n distinct eigenvalues, it is diagonalizable (5.6).
- * Similar matrices have the same eigenvalues (5.4).
- The dimension of an eigenspace is at least one and at most the multiplicity of the corresponding eigenvalue (5.7).
- Every matrix has n eigenvalues over $\mathbb{C}$ (with multiplicity).
- * Pythagorean theorem (6.2).
- * The orthogonal complement of a subspace is a subspace.
- * An orthogonal set of nonzero vectors is linearly indep (6.4).
- * Coefficients with respect to an orthogonal basis (6.5).

- * Decomposition theorem (6.8).
- Best approximation theorem (6.9).
- Formula for orthogonal projection given orthonormal basis(6.10).
- Gram-Schmidt (6.11).
- Least Squares (6.13).

Algorithms.

- Row reduction (obviously) for: linear system solving, finding bases for null and column spaces, finding rank, finding eigenvectors, checking linear independence/span, determinant computation.
- Determinant computation by recursive formula involving submatrices.
- Gram-Schmidt.
- Least Squares via Normal Equations.

Types of Problems. The problems will be similar to the homework problems, but to help you study, here are the main kinds:

- Check whether something is a vector space, subspace, linear transformation.
- Find the matrix of a linear transformation with respect to a pair of bases.
- Find the coordinates of a vector with respect to a basis, or change coordinates from one basis to another.
- Decide whether a linear transformation is 1-1/onto/isomorphism. Find bases for kernel and image using the correspondence principle.
- Find the dimension of a subspace. Find a basis for a subspace.
- Compute the determinant of a matrix.
- Find the eigenvalues/eigenvectors of a matrix.
- Figure out whether a matrix is diagonalizable, if so possibly diagonalize it, possibly using complex numbers.
- Decide whether two matrices are similar.
- Use diagonalization to compute a high power of a matrix.
- Find the orthogonal projection of a vector onto a subspace. Decompose a vector into components in W and $W^\perp$.
- Find a basis / orthogonal basis for a subspace or its orthogonal complement.
- Find an example of a matrix with some property.
- True/false conceptual questions.
- Prove that something is true, often requiring concepts from multiple topics (e.g., Problem 6 on the practice midterm).
- State a definition or prove a theorem from class (see above).

This list is not comprehensive. I will most likely give you problems that involve combining ideas from multiple chapters.

> **SHOW YOUR WORK. If you just write a numerical answer without a clear description of the process you used to get it, you will only get partial credit. If something really is obvious (such as saying that the vector $(1,1)$ is orthogonal to $(1,-1)$) use the words "observe that" or "by inspection" to indicate this.**

Not on the exam QR factorization will NOT be on the exam, since we did not cover it in class or on the HW.