1. True or False (no need for justification):

(a) If $V$ is a vector space with a finite basis then $V$ is isomorphic to $\mathbb{R}^n$ for some $n$.
(b) The system $A^T Ax = A^T b$ is consistent for all $A$ and $b$.
(c) If $A$ and $B$ are similar and $A$ is diagonalizable then $B$ must be diagonalizable.
(d) The rank of a square matrix is equal to the number of nonzero eigenvalues (counted with multiplicity).
(e) If $x$ and $y$ are arbitrary nonzero vectors in $\mathbb{R}^n$ then there is a basis $B$ of $\mathbb{R}^n$ such that $[x]_B = y$.
(f) Every eigenvalue of a square matrix $A$ is a pivot of $A$ in the reduced row echelon form of $A$.
(g) If $A$ is a square matrix then $A$ and $A^T$ have the same eigenvalues.
(h) An upper triangular matrix is always diagonalizable.
(i) An set of orthogonal vectors is always linearly independent.
(j) If $v_1, \ldots, v_k \in \mathbb{R}^n$ are linearly independent and $W$ is a subspace of $\mathbb{R}^n$ then $\text{Proj}_W(v_1), \ldots, \text{Proj}_W(v_k)$ must also be linearly independent.

2. Let $V = \mathbb{R}^{3\times3}$ denote the vector space of real $3 \times 3$ matrices with addition and scalar multiplication defined entrywise. Let

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

and consider the subset

$$W = \{ X \in V : XM = MX \}$$

of matrices in $V$ which commute with $M$. Is $W$ a subspace of $V$? If so, prove it. If not, explain why.
3. Let \( P_2 = \{a_0 + a_1 t + a_2 t^2\} \) be the vector space of polynomials of degree at most 2 with coefficient-wise operations, and consider the linear transformation \( T : P_2 \to P_2 \) defined by

\[
T(q) = \frac{d^2}{dt^2}q + t \cdot \frac{d}{dt}q + 3q.
\]

Is there a basis \( B \) of \( P_2 \) such that the matrix of \( T \) with respect to \( B \) is diagonal? If so, find such a basis as well as the corresponding matrix. If not, explain why.

4. Let \( A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \). Find the eigenvalues of \( A \). Compute \( A^{11} \).

5. Consider the vectors

\[
x_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \quad x_2 = \begin{bmatrix} 12 \\ -4 \\ 7 \end{bmatrix} \quad y = \begin{bmatrix} 2 \\ 4 \\ 0 \\ -1 \end{bmatrix},
\]

and let \( W = \text{Span}\{x_1, x_2\} \). Find vectors \( w \) and \( z \) such that \( w \in W, z \in W^\perp \), and \( y = w + z \). What is the distance between \( y \) and the closest point in \( W \) to \( y \)?

6. Let \( W \) be a subspace of \( \mathbb{R}^n \) and let \( P \) be the standard matrix of the projection onto \( W \), i.e., \( P = [\text{Proj}_W] \) where \( \text{Proj}_W : \mathbb{R}^n \to \mathbb{R}^n \) is the linear transformation which projects onto \( W \). (a) Show that \( P^2 = P \). (b) Use this to show that the eigenvalues of \( P \) are all either 0 or 1. (c) What is the eigenspace corresponding to the eigenvalue 1 of \( P \)?