Math 54 Fall 2016 Practice Midterm 1

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50 minutes, closed book, closed notes

1. (20 points) True or False (no need for justification):
   (a) If the reduced row echelon form of the augmented matrix of a linear system has a column containing only zeros, then it must be consistent.
   (b) If the columns of A are linearly independent, then $Ax = b$ is consistent for every b.
   (c) If A has linearly dependent columns, then $Ax = 0$ is has infinitely many solutions.
   (d) If A is an $m \times n$ matrix and $b \in \mathbb{R}^m$, then the set of solutions to $Ax = b$ is a linear subspace of $\mathbb{R}^n$.
   (e) If a linear subspace of $\mathbb{R}^n$ contains more than one vector, then it must contain infinitely many vectors.
   (f) If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation then T cannot be both $1-1$ and onto.
   (g) If A is an $m \times n$ matrix and B is an $n \times p$ matrix whose first column contains only zeros, then the first column of AB also contains only zeros.
   (h) If A, B, C are invertible then the product ABC must also be invertible.
   (i) If A is a square matrix such that $A^2 = 0$ then $A = 0$.
   (j) If A and B are square matrices of rank 2 then the product AB has rank at most 2.

2. (8 points) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Find a 3×2 matrix B such that $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

3. (11 points) Consider the vectors

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 3 \\ 0 \\ 3 \\ 6 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, v_5 = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 6 \end{bmatrix}.$$

   (a) Let $H = \text{span}\{v_1, v_2, v_3, v_4, v_5\}$. Find a subset of the given vectors which forms a basis for H.

(b) What is the dimension of H?

(c) Determine whether the vector

$$w = \begin{bmatrix} 2 \\ 0 \\ 3 \\ 4 \end{bmatrix}$$

lies in H.

4. (12 points) Let $T_1 : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by

$$T_1\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 + x_3 \end{bmatrix},$$

and let $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which rotates a vector about the origin by $\pi/4$ radians counterclockwise.

(a) Determine the standard matrix of the composition $T_2 \circ T_1 : \mathbb{R}^3 \to \mathbb{R}^2$ defined by applying T_1 and then T_2.

(b) Determine whether $T_2 \circ T_1$ is onto.

(c) Determine whether $T_2 \circ T_1$ is one to one.

5. (9 points)

(a) Give an example of a 3×3 matrix whose null space has dimension 1.

(b) Give an example of a 3×3 matrix whose column space has dimension 1.

(c) Is there a 3×3 matrix whose null space and column space *both* have dimension 1?