Math 54 First Midterm Exam, Prof. Srivastava
September 23, 2016, 4:10pm–5:00pm, 155 Dwinelle Hall.

Name: ______________________________

SID: ______________________________

Instructions: Write all answers in the provided space. This exam includes two pages of scratch paper, which must be submitted but will not be graded. Do not under any circumstances unstaple the exam. Write your name and SID on every page. Show your work — numerical answers without justification will be considered suspicious.

Calculators, phones, cheat sheets, textbooks, and your own scratch paper are not allowed.

UC Berkeley Honor Code: *As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.*

Sign here: ______________________________

Question	Points
1	20
2	12
3	10
4	18
Total:	60

Do not turn over this page until your instructor tells you to do so.

Name and SID: ______________________________

1. (20 points) Circle always true (**T**) or sometimes false (**F**) for each of the following. There is no need to provide an explanation. Two points each.

   For parts (a)-(f), assume A is an arbitrary $m \times n$ matrix and R is the reduced row echelon form of A. For parts (g) and (h) assume A and B are arbitrary $n \times n$ square matrices.

   (a) A is invertible if and only if R is invertible. **T** **F**

   (b) The column space of A is equal to the column space of R. **T** **F**

   (c) The null space of A is equal to the null space of R. **T** **F**

   (d) The linear transformations $T(\mathbf{x}) = A\mathbf{x}$ and $S(\mathbf{x}) = R\mathbf{x}$ are the same. **T** **F**

   (e) $\mathrm{Col}(A) = \mathbb{R}^m$ if and only if $\mathrm{Col}(R) = \mathbb{R}^m$. **T** **F**

   (f) If $\mathbf{b}_1, \ldots, \mathbf{b}_n$ is a basis for $\mathbb{R}^n$, then $A\mathbf{b}_1, \ldots, A\mathbf{b}_n$ must be a basis for $\mathrm{Col}(A)$. **T** **F**

   (g) If A and B are invertible then $A + B$ must be invertible. **T** **F**

   (h) If B is invertible and every entry of A is greater than or equal to the corresponding entry of B, then A must be invertible. **T** **F**

   (i) Every invertible matrix can be written as a product of elementary matrices. **T** **F**

   (j) The set of vectors in $\mathbb{R}^4$ whose entries have a nonnegative sum: i.e.

$$\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} : x_1 + x_2 + x_3 + x_4 \geq 0 \right\}$$

   is a linear subspace of $\mathbb{R}^4$. **T** **F**

2. For each of the following, find a pair of 2×2 matrices A and B with the specified property, or explain why no such matrices exist.

   (a) (4 points) $AB \neq BA$.

   (b) (4 points) AB is invertible but BA is not invertible.

   (c) (4 points) $\operatorname{rank}(A) + \operatorname{rank}(B) = \operatorname{rank}(A + B)$ and $A \neq 0, B \neq 0$.

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[Scratch Paper 1]

3. (10 points) Consider the vectors:

$$\mathbf{v}_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 9 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 5 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 2 \\ -4 \\ 7 \\ -7 \end{bmatrix}.$$

Find a vector from $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ which is in the span of the remaining vectors. Express this vector as a linear combination of the remaining ones.

4. Consider the linear transformation $T_1 : \mathbb{R}^3 \to \mathbb{R}^2$ defined by:

$$T_1\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_2 + x_3 \end{bmatrix}$$

and let $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates a vector counterclockwise by $\pi/2$ radians.

(a) (5 points) Let $T = T_2 \circ T_1$, $T : \mathbb{R}^3 \to \mathbb{R}^2$ denote the composition of T_1 and T_2. Find the standard matrix of T. Call this matrix A.

(b) (5 points) Find bases for $\mathrm{Col}(A)$ and $\mathrm{Null}(A)$.

(c) (2 points) Is T onto? Explain why in terms of your answers to part (b).

(d) (6 points) Find two **distinct** vectors $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^3$ such that $T(\mathbf{v}_1) \neq 0$, $T(\mathbf{v}_2) \neq 0$, and $T(\mathbf{v}_1) = T(\mathbf{v}_2)$.

[Scratch Paper 2]