Hw 6 Solution

Math 54

0.1 Section 4.3

26. Note that $sin\,t\,cos\,t = \frac{1}{2}sin\,2t$, $sin\,t$ and $sin\,2t$ are linearly independent. So a basis is given by $\{sin\,t, sin\,2t\}$.

31. If $\{v_1, \cdots, v_p\}$ are linearly dependent, then for some nonzero $\{c_i\}$, $c_1v_1 + \cdots + c_pv_p = 0$, which imply $c_1T(v_1) + \cdots + c_pT(v_p) = 0$.

32. If $\{Tv_1, \cdots, Tv_p\}$ are linearly dependent, then for some nonzero $\{c_i\}$, $c_1T(v_1) + \cdots + c_pT(v_p) = 0$, so $T(c_1v_1 + \cdots + c_pv_p) = 0$. Since T is one to one, $c_1v_1 + \cdots + c_pv_p = 0$, so $\{v_1, \cdots, v_p\}$ are linearly dependent

33. $\{p_1, p_2\}$ are linearly independent. Because they are not multiples of each other.

33. $p_1 + p_2 - p_3 = 0$. So the three polynomials are linear dependent. $\{p_1, p_2\}$ form a basis of Span$\{p_1, p_2, p_3\}$

37. Suppose for some $\{c_i\}$, $c_1t + c_2sin\,t + c_3cos2t + c_4sin\,t\,cos\,t = 0$. Let $t = 0, \pi/4, \pi/2, \pi$, we get four linear equations.

$$c_3 = 0$$
$$\pi c_1 + 2c_2 - 2c_3 = 0$$
$$\pi c_1 + c_3 = 0$$
$$\pi c_1 + 2\sqrt{2}c_2 + 2c_4 = 0$$

The coefficient matrix is invertible, so we only have trivial solution. So the four functions are linearly independent.

0.2 Section 4.2

30. $cT(x) + T(w) = T(cx + w) \in Range(T)$. So $Range(T)$ is a subspace of W.

31. (a)

$$T(cp + q) = \begin{bmatrix} cp(0) + q(0) \\ cp(1) + q(1) \end{bmatrix} = c\begin{bmatrix} p(0) \\ p(1) \end{bmatrix} + \begin{bmatrix} q(0) \\ q(1) \end{bmatrix} = cT(p) + T(q)$$

So T is linear transformation.
(b)Suppose $p(t) = a + bt + ct^2 \in Ker(T)$, we have

$$T(p) = \begin{bmatrix} a \\ a + b + c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So $p(t) = b(t - t^2)$. $Ker(T) = Span\{t - t^2\}$, a basis is $\{t - t^2\}$.
$dim\ Im(T) = dim\mathbb{P}_2 - dim\ Ker(T) = 3 - 1 = 2$ So $Range(T) = \mathbb{R}^2$.

35. $cT(x) + T(w) = T(cx + w) \in T(U)$ for $x, w \in U$. So $T(U)$ is a subspace of W.

36. If $T(x) \in Z, T(w) \in Z$, then $T(cx + w) = cT(x) + T(w) \in Z$. So U is a subspace of V.

0.3 Section 4.4

13. $p(t) = 1 + 4t + 7t^2 = 2(1 + t^2) + 6(t + t^2) - (1 + 2t + t^2)$, the coordinate is

$$\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}$$

14. $p(t) = 1 + 3t - 6t^2 = 3(1 - t^2) + 2(t - t^2) - (2 - t + t^2)$, the coordinate is

$$\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

18. Because $b_1 = 1 \cdot b_1 + 0 \cdot b_2 + \cdots 0 \cdot b_n$, so the coordinate of b_1 is e_1. Similar for other b_i.

19. Clearly, S spans V. To show vectors in V are linearly independent, suppose $c_1 v_1 + \cdots c_p v_p = 0$. Since 0 has a unique representation as a linear combination of elements of S, so $c_i = 0$.

20. Suppose $c_1 v_1 + \cdots c_4 v_4 = 0$. If $w = k_1 v_1 + \cdots + k_4 v_4$, then $w = (c_1 + k_1)v_1 + \cdots + (c_4 + k_4)v_4$. So w has a different representation.

22. Let A be a matrix whose column vectors are $b_1, \cdots b_n$. Then the coordinate map is given by the matrix $A^(-1)$.

23. If $[u]_B = [v]_B$, then the representation of u, v in basis B are the same, so $u = v$.

24. Let $u = y_1 b_1 + \cdots + y_n b_n$, then $[u]_B = y$.

26. Since coordinate map is a linear isomorphism, the linear combination is preserved by linear isomorphism.

28. The coordinate vectors are

$$\begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

They are linearly independent by row reduction.

31. (a)The coordinate vectors are

$$\begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ -7 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -6 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

4 vectors in $\mathbb{R}^3$ must be linearly dependent.

(b)The coordinate vectors are

$$\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -8 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$$

4 vectors in $\mathbb{R}^3$ must be linearly dependent.

32. (a)The coordinate vectors are

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$$

They are linearly independent and span $\mathbb{R}^3$, so they form a basis.
(b)$q(t) = -(1 + t^2) + 1(t - 3t^2) + 2(1 + t - 3t^2) = 1 + 3t - 10t^2$

0.4 Section 4.5

21. The coordinate vectors are

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -12 \\ 0 \\ 8 \end{bmatrix}$$

The 4 vectors are linearly independent and span $\mathbb{P}_3$.

23. $p(t) = -1 + 8t^2 + 8t^3 = 1(8t^3 - 12t) + 2(4t^2 - 2) + 6(2t) + 3$. So the coordinate is

$$\begin{bmatrix} 3 \\ 6 \\ 2 \\ 1 \end{bmatrix}$$

0.5 Section 5.4

2. The matrix form is

$$\begin{bmatrix} 3 & -2 \\ -3 & 5 \end{bmatrix}$$

6. (a)$T(p(t)) = p(t) + 2t^2 p(t) = 3 - 2t + 7t^2 - 4t^3 + 2t^4$.
(b)$T(cp + q) = cp(t) + q(t) + 2t^2(cp(t) + q(t) = c(p(t) + 2t^2 p(t)) + q(t) + 2t^2 q(t) = cT(p) + T(q)$
(c)The matrix is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

9. (a)

$$T(p) = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

(b)

$$T(cp + q) = \begin{bmatrix} cp(-1) + q(-1) \\ cp(0) + q(0) \\ cp(1) + q(1) \end{bmatrix} = \begin{bmatrix} cp(-1) \\ cp(0) \\ cp(1) \end{bmatrix} + \begin{bmatrix} q(-1) \\ q(0) \\ q(1) \end{bmatrix} = cT(p) + T(q)$$

(c)

$$T(a + bt) = \begin{bmatrix} a - b \\ a \\ a + b \end{bmatrix}$$

The matrix is

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

0.6 Ch 4 supplementary

4. t is not a scalar, so they are not linearly dependent.

5. The coordinate vectors are

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The 1st, 2nd, 4th vectors form pivot columns of the coefficent matrix, so p_1, p_2, p_4 is a basis.

6. If two of them are not multiple of each other, then these two form a basis.
8. Since they have same dimension, a basis of H is also a basis of V. So $H = V$