MATH 53 SPRING 2018 MIDTERM 1 LIST OF TOPICS

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The first midterm will be based on the first ten lectures of the class, except for partial differential equations. Here is a list[1] of things you should know / know how to do; if you are comfortable with everything on this list, you should be fine.

The emphasis will be on computational problems, conceptual questions, and occasionally visualization, and not on formal proofs. However, you should know all of the definitions and informal proofs that were covered in lecture.

- Know the distance formula for Euclidean geometry in two and three dimensions,
- For each of the vector operations (length, addition, multiplication by scalars, dot product, determinant, cross product), compute it and understand its geometric meaning.
- Algebraically manipulate expressions involving vector operations, avoiding undefined operations (e.g. the sum of a vector and a scalar).
- Know how to find the projection of one vector along another vector.
- Write the equation for a line in parametrized form, given a point on the line and a tangent vector to the line.
- Write the equation for the line or line segment between two points.
- Write the equation of a plane in the form $ax + by + cz + d = 0$ given a point on the plane and a normal vector, and vice versa.
- Find the equation of the plane through three points.
- Understand what a parametrized curve is, as opposed to just a curve.
- Understand how to write parametrized curves in space as vector-valued functions.
- Write down a formula for a parametrized curve from a description.
- Calculate the length of a parametrized curve.
- Compute the tangent line to a parametrized curve at a point.
- Compute the length of a parametrized curve in space.
- Compute the velocity and speed of a parameterized curve at a point.
- Know how to reparameterize a curve in terms of arc length.
- Know the rules of calculus for vector-valued functions, namely three versions of the product rule and an analogue of the fundamental theorem of calculus.
- Sketch the graph of a function of two variables. Highly accurate drawings are not required.
- Sketch the level sets of a function of two or three variables. Understand how some properties of a function are reflected in the appearance of its level sets.
- Understand the different ways that a given surface or curve might be presented (i.e., as the set of points satisfying some equation, as the graph of a function, or as a parameterized curve or surface), and how to switch between them.
- Plot traces (obtained by fixing one variable when a surface is given by equations) and projections (obtained by ignoring one variable when a curve is given as a parameterized equation) as well as other visualization heuristics such as plotting tangent directions, discussed in lecture.
- Derive the parametric equation of the trajectory of a particle given a verbal description, by decomposing the motion into simpler motions using vectors.

[1] This list is a modified version of a similar list produced by Prof. Hutchings for a previous version of this course.

- Know the basic limit properties and the definition of a continuous function.
- Use limit properties and continuity to compute limits when they exist.
- Prove in some cases that the limit of a function as (x, y) ? (a, b) does not exist by considering the limits along different curves approaching (a, b).
- Know the definition of partial derivatives.
- Compute partial derivatives.
- Know Clairauts theorem.
- Compute partial derivatives of functions that are defined implicitly in basic examples.
- Determine the tangent plane to the graph of a function of two variables at a point.
- Determine the tangent plane to the level surface of a function of three variables at a point.
- Understand that a function is differentiable at a point when the graph is well approximated by the tangent plane.
- Use linear approximation to approximate the value of functions of two and three variables.
- Perform calculations using the different versions of the chain rule in multivariable calculus, and know where to evaluate the various functions that appear in the formulas.
- Calculate partial derivatives of functions defined implicitly in the general case, using the total differential and the chain rule.
- Know the definition of directional derivatives, total differential, and gradient, and the relationship between them.
- Know the geometric interpretation of the gradient, namely that it is perpendicular to the level sets and points in the direction in which the directional derivative is largest.
- Know the vector version of multivariable chain rule.
- Use the gradient to find the tangent line/plane to a level set of a function of two or three variables.
- Know the definitions of local and global minima and maxima, and critical points.
- Know the second derivative test for functions of two variables.
- Know the statement of the Extreme Value Theorem and understand why the hypotheses of closed, bounded, and continuous are necessary.
- Find global minima and maxima of a function on a domain by looking for critical points (and points where the partial derivatives are not both defined) and minima and maxima on the boundary.
- Understand why the method of Lagrange multipliers works.
- Solve constrained optimization problems using Lagrange multipliers.
- Know the significance of the Lagrange multiplier.