Math 53 Spring 2018 Practice Midterm 2

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80 minutes, closed book, closed notes

1. Calculate
$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)^{2018} dxdy$$

2. Find the volume of the region enclosed by the plane $z = 4$ and the surface $z = (2x-y)^2 + (x+y-1)^2$ (hint: change of variables)

3. Suppose E is the region in space bounded by the surface
$$z = 1 - y^2$$
and the planes $x + z = 1, x = 0$, and $z = 0$. (a) Set up (but do not evaluate) an iterated integral for the mass of a solid with shape E and density $\rho(x, y, z) = xyz$ in the order $dxdydz$. (b) Set up (but do not evaluate) an integral for the same quantity but in the order $dzdxdy$.

4. Find the volume of the part of the ball $\rho \leq a$ that lies between the cones $\phi = \pi/6$ and $\phi = \pi/3$, where all regions are given in spherical coordinates.

5. Consider the force field $\mathbf{F} =< x, x >$. Let C be the curve consisting of a straight line segment from $P = (-1, 0)$ to $Q = (0, 1)$ with unit speed, and then another from $Q = (0, 1)$ to $R = (1, 0)$ with unit speed. (a) Find the work done by $\mathbf{F}$ on a particle moving along C. (b) Find a curve connecting P and R along which the work done is zero.

6. Find the values of a and b for which the vector field
$$\mathbf{F} =< axy^2 + x^2, x^2y + bx >,$$
is conservative. For these a and b, find a potential f such that $\mathbf{F} = \nabla f$.

7. True or False:

   (a) The line integral of a vector field $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of the direction of the curve C (i.e., does not change if the direction is reversed).

(b) If F is a vector field such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ and $\int_C \mathbf{F} \cdot \mathbf{n} ds = 0$ for every simple closed curve C in the plane, then $\mathbf{F}$ must be constant.

(c) If $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for *every* curve C in the plane, then $\mathbf{F}$ must be identically zero.

8. Consider the parameterized curves:

$$C_1 : \mathbf{r}(t) =< t, \sin(3t) >, t \in [0, \pi]$$

$$C_2 : \mathbf{r}(t) =< \pi/2 + (\pi/2)\cos(t), (\pi/2)\sin(t) >, t \in [0, \pi].$$

Let $C := C_1 + C_2$ (i.e., C_1 followed by C_2). (a) Draw a sketch of C. (b) Use Green's theorem applied to an appropriate vector field to find the area enclosed by C. (c) Let $\mathbf{F} =< x + y + \pi, x + y + 2\pi >$. What is the flux $\int_C \mathbf{F} \cdot \mathbf{n} ds$ of F across C?