Math 53 Spring 2018 Practice Midterm 1

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80 minutes, closed book, closed notes

1. Consider the parameterized curve defined by:

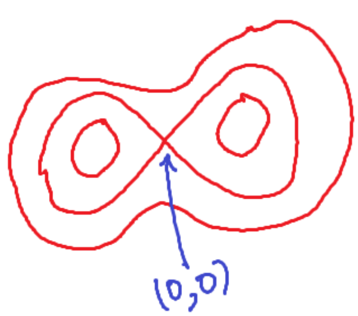
$$r(t) = \langle t^3 - 1, t^2 + 2t, \ln(t) - 1/t \rangle.$$

Find the cosine of the angle between the vector $\overline{v} = \langle 1, 2, 2 \rangle$ and the tangent vector to this curve at the point of its intersection with the yz plane.

2. A ladybug is climbing on a Volkswagen Bug (= VW). In its starting position, the the surface of the VW is represented by the unit semicircle $x^2 + y^2 = 1, y \geq 0$ in the xy-plane. The road is represented as the x-axis. At time $t = 0$ the ladybug starts at the front bumper, $(1, 0)$, and walks counterclockwise around the VW at unit speed relative to the VW. At the same time the VW moves to the right at speed 10. a) Use vector notation to find the parametric formula for the trajectory of the ladybug, and find its position when it reaches the rear bumper. (At $t = 0$, the rear bumper is at $(-1, 0)$.) b) Compute the speed of the bug, and find where it is largest and smallest. Hint: It is easier to work with the square of the speed.

3. Consider the function $f(x, y, z) = x^2 + y^2/4 + z^2/9$. (a) Sketch the $x = 1/2$ and $y = 0$ traces of the level surface $S = \{(x, y, z) : f(x, y, z) = 1$, labeling intersections with the coordinate axes. (b) Find a point P such that the tangent plane to S at P is perpendicular to the vector $\langle 1, 1, 1 \rangle$. (c) Does the plane you found in part (b) pass through the origin? Explain why or why not.

4. Suppose the following is a contour plot of a differentiable function $f(x, y)$:



Explain why $(0,0)$ must be a critical point of f.

5. The surfaces $x^2+y^2=2$ and $y=z$ intersect in a curve C. Find a parametric equation of the tangent line to this curve passing through the point $(1,1,1)$.

6. If $z=xe^{-y}$, $x=\sin(s+t)$, and $y=\cos(s-t)$, find $\partial z/\partial s$ and $\partial z/\partial t$.

7. Write down the total differential of the function

$$f(x,y,z)=\sqrt{x^2+4y^2+z^2}$$

at $(3,1,6)$ and use it to find an approximation of the number

$$\sqrt{(3.02)^2+(0.99)^2+(5.97)^2}.$$

What is the directional derivative of f at the point $(3,1,6)$ in the direction $\overline{u}=(\hat{i}-\hat{j}+\hat{k})/\sqrt{3}$?

8. Find the critical points of the function

$$f(x,y)=x^4+2y^2-4xy$$

and classify each as a local minimum, maximum, or saddle point.

9. Use Lagrange multipliers to find the point on the plane $2x+y-z=6$ which is closest to the origin. (hint: use the squared distance)

10. Suppose $x^2+y^3-z^4=1$ and $z^3+zx+xy=3$. (a) Take the total differential of each of these equations. (b) The two surfaces in part (a) intersect in a curve along which y is a function of x. Find dy/dx at $(1,1,1)$.