Math 53 Second Midterm Exam, Prof. Srivastava
April 12, 2018, 5:10pm–6:30pm, 155 Dwinelle Hall.

Name: Nikhil Srivastava

SID: ______

GSI: ______

Name of the student to your left: ______
Name of the student to your right: ______

Instructions: Write all answers clearly in the provided space. This exam includes some space for scratch work at the bottom of pages 2 and 4 which will not be graded. Do not under any circumstances unstaple the exam. Write your name and SID on every page. **Show your work** — numerical answers without justification will be considered suspicious and will not be given full credit. Calculators, phones, cheat sheets, textbooks, and your own scratch paper are not allowed.

UC Berkeley Honor Code: *As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.*

Sign here: ______

Question	Points
1	14
2	14
3	12
4	8
5	14
6	12
7	12
8	14
Total:	100

Do not turn over this page until your instructor tells you to do so.

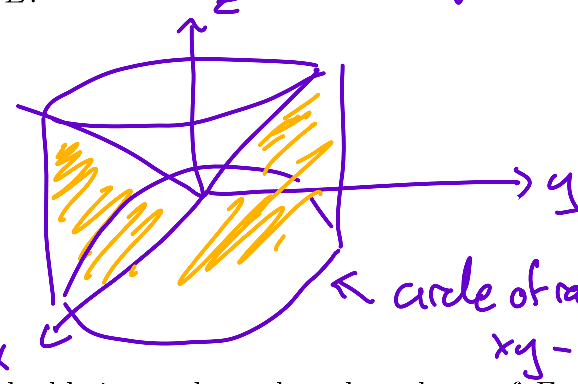
Name and SID: ______________________________

1. Let E be the solid region bounded from above by the surface $z^2 = x^2 + y^2$, from below by the plane $z = 0$, and from the sides by the surface $x^2 + y^2 = 1$.

cone

↳ cylinder of radius 1

(a) (4 points) Draw a rough sketch of E.



(b) (10 points) Set up and evaluate a double integral equal to the volume of E.

Vol(E) = volume under the graph of $z = \sqrt{x^2+y^2}$ above the domain $D = \{(x,y) : x^2+y^2 \le 1\}$.

$$= \iint_D \sqrt{x^2+y^2}\, dA.$$

Since D and the function are radially symmetric, polar coordinates should help.

$$D = \{(r,\theta) : 0 \le \theta \le 2\pi,\ 0 \le r \le 1\}$$

$$\sqrt{x^2+y^2} \longrightarrow \sqrt{r^2\cos^2\theta + r^2\sin^2\theta} = r$$

$$dA \longrightarrow r\,dr\,d\theta$$

So $\text{Vol} = \int_0^{2\pi}\int_0^1 r \cdot r\, dr\, d\theta = \int_0^{2\pi} \frac{r^3}{3}\Big|_0^1 d\theta = \frac{1}{3}\cdot 2\pi = 2\pi/3$

2. (14 points) Let R be the parallelogram with vertices $(0,0), (1,1), (2,-1)$, and $(3,0)$. Use the change of variables

$$x = u + 2v, \quad y = u - v$$

to evaluate the integral

$$\int\int_R (x+2y)^2 e^{x-y} dA.$$

3. (12 points) Find the average distance from a point in a ball of radius a (i.e., a solid sphere in $\mathbb{R}^3$ centered at the origin) to its center.

Name and SID: ____

4. (8 points) Change the order of integration:

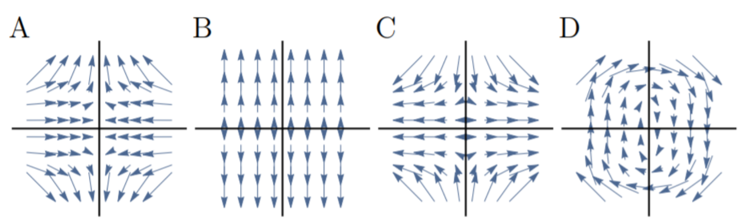
$$\int_0^1 \int_0^{\sqrt{z}} \int_0^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz = \int_?^? \int_?^? \int_?^? f(x,y,z)\,dy\,dz\,dx.$$

5. (a) (6 points) Let E be the region in $\mathbb{R}^3$ bounded by the sphere of radius 3 at the origin, the sphere of radius 4 at the origin, the cone $z = -\sqrt{x^2+y^2}$, and the cone $z = -2\sqrt{x^2+y^2}$. Which of the following integrals represents the volume of E? (circle exactly one)

   1. $\int_3^4 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{x^2+y^2}}^{-2\sqrt{x^2+y^2}} dz\,dy\,dx$
   2. $\int_3^4 \int_0^{2\pi} \int_{3\pi/4}^{5\pi/6} d\phi\,d\theta\,d\rho$
   3. $\int_3^4 \int_0^{2\pi} \int_{-2r^2}^{-r^2} r\,dz\,dr\,d\theta$
   4. None of the above.

   (b) (8 points) Match the vector field to the picture. There is an exact match.

Field	A-D
$\vec{F}(x,y) = \langle 0, y\rangle$	
$\vec{F}(x,y) = \langle -x, y^3\rangle$	
$\vec{F}(x,y) = \langle y^3, -x\rangle$	
$\vec{F}(x,y) = \langle x, -y^3\rangle$	



Name and SID: ______________________________

[Scratch Paper 1]

6. Consider the vector field $\mathbf{F} = \langle 4x\ln(y), \frac{2x^2-1}{y}\rangle$ defined on $D = \{(x,y) : y > 0\}$.

   (a) (4 points) Explain why $\mathbf{F}$ is conservative.

   (b) (8 points) Using a systematic method, find a potential $f(x,y)$ defined on D such that $\mathbf{F} = \nabla f$.

7. (12 points) Calculate the work done by the force field $\mathbf{F} = \langle y^2 + 1, 2xy + \sin^6(y) \rangle$ on a particle moving in the plane with trajectory $\mathbf{r}(t) = \langle e^t, (t^6 - 1)\sin(t) \rangle$, $t \in [0, 1]$.

8. Let $\mathbf{F} = \langle 2xy, 3xy \rangle$, and let C be a positively oriented unit circle centered at the origin. Use Green's theorem to evaluate:

   (a) (7 points) The work $\int_C \mathbf{F} \cdot d\mathbf{r}$.

   (b) (7 points) The flux $\int_C \mathbf{F} \cdot \mathbf{n} ds$.

Name and SID: ______________________________

[Scratch Paper 2]