

MATH 53 FIRST MIDTERM EXAM, PROF. SRIVASTAVA
FEBRUARY 22, 2018, 5:10PM–6:30PM, 155 DWINELLE HALL.

Name: Nikhil Srivastava

SID: _____

GSI: _____

NAME OF THE STUDENT TO YOUR LEFT: _____

NAME OF THE STUDENT TO YOUR RIGHT: _____

INSTRUCTIONS: Write all answers clearly in the provided space. This exam includes some space for scratch work at the bottom of pages 2 and 4 which will not be graded. Do not under any circumstances unstaple the exam. Write your name and SID on every page. Show your work — numerical answers without justification will be considered suspicious and will not be given full credit. Calculators, phones, cheat sheets, textbooks, and your own scratch paper are not allowed.

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Sign here: _____

Question	Points
1	10
2	10
3	13
4	10
5	21
6	10
7	10
8	16
Total:	100

Do not turn over this page until your instructor tells you to do so.

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1. (10 points) Find a parametric equation of the line defined by the intersection of the planes:

$$x - y + 3z = 5$$

$$3x + y - z = 3.$$

Let's choose y as the parameter since it appears simply in both equations. We then have:

$$x + 3z = 5 + t$$

$$3x - z = 3 - t$$

$$4x + 2z = 8 \implies z = 4 - 2x$$

$$\implies 3x - (4 - 2x) = 3 - t$$

$$\implies 5x = 7 - t \implies x = \frac{7}{5} - \frac{t}{5}$$

$$\implies z = 4 - \frac{14}{5} + \frac{2t}{5} = \frac{2t}{5} + \frac{6}{5}$$

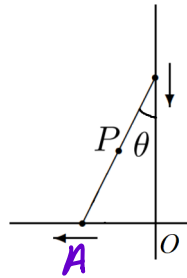
[Scratch Space Below]

So an equation is:

$$\underline{\underline{\vec{r}(t) = \left\langle -\frac{t}{5} + \frac{7}{5}, t, \frac{2t}{5} + \frac{6}{5} \right\rangle}}$$

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2. The top extremity of a ladder of unit length rests against a vertical wall, while the bottom is being pulled away, as shown below.



- (a) (8 points) Find a parameterized curve $\mathbf{r}(\theta)$ tracing the trajectory of the midpoint P of the ladder as it goes from fully vertical to horizontal, using as parameter the angle θ between the ladder and the vertical wall, and treating the point at which the wall and the ground meet as the origin.

By trigonometry, because the hypotenuse has length one we have:

$$|\vec{OA}| = \sin(\theta) \implies \vec{OA} = \langle -\sin\theta, 0 \rangle$$

$$\vec{AP} = \frac{1}{2} \langle \sin\theta, \cos\theta \rangle \text{ since } |\vec{AP}| = \frac{1}{2}$$

$$\begin{aligned} \text{Thus, the point } P \text{ has position vector } \vec{OP} &= \vec{OA} + \vec{AP} \\ &= \langle -\frac{1}{2}\sin\theta, \frac{1}{2}\cos\theta \rangle \end{aligned}$$

$$\text{and the param. curve is } \vec{r}(\theta) = \langle -\frac{1}{2}\sin\theta, \frac{1}{2}\cos\theta \rangle$$

- (b) (2 points) Is the speed of P (as a function of θ) increasing, decreasing, or constant as θ varies from 0 to $\pi/2$?

$$\text{The velocity is } \vec{r}'(\theta) = \langle -\frac{1}{2}\cos\theta, -\frac{1}{2}\sin\theta \rangle$$

$$\begin{aligned} \text{So the speed is } |\vec{r}'(\theta)| &= \sqrt{\frac{1}{4}\cos^2\theta + \frac{1}{4}\sin^2\theta} \\ &= \frac{1}{2}, \text{ which is constant.} \end{aligned}$$

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3. Consider the function $f(x, y) = xy$.

(a) (5 points) Roughly sketch the level curves $f(x, y) = 1$, $f(x, y) = 2$, and $f(x, y) = -1$, labeling which is which.

(b) (5 points) Find a point P on the curve $f(x, y) = 1$ at which the directional derivative along the direction $\mathbf{u} = \frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}$ is equal to zero.

(c) (3 points) Sketch the direction of the vector ∇f at the point P that you found above, originating from P in the drawing in part (a).

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4. (10 points) The parameterized curves $\mathbf{a}(t) = \langle 3t, t^2 - 2, e^t + 1 \rangle$ and $\mathbf{b}(s) = \langle s^2 - s, -2s, s^3 + s^2 \rangle$ intersect at the point $P = (0, -2, 2)$. Find the cosine of the (acute) angle of their intersection at P .

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5. Consider the function $f(x, y) = \sqrt{e^x + 3e^{y-1}}$.

(a) (5 points) Find the total differential of f at $x = 0, y = 1$.

(b) (5 points) Find the equation of the tangent plane to the graph $\{(x, y, z) : z = f(x, y)\}$ of f at the point $P = (0, 1, f(0, 1))$.

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(c) (5 points) Use the equation from (b) to compute an approximation to the value of $f(.01, .99)$.

(d) (3 points) Is there a point Q on the graph at which the tangent plane to the graph is horizontal (i.e., parallel to the xy -plane)? If so, find such a point, and if not explain why.

(e) (3 points) Is there a point R on the graph where the tangent plane is vertical (i.e., perpendicular to the xy -plane)? If so, find such a point, and if not explain why.

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6. (10 points) Let $w = w(x, y)$ be a differentiable function and let $x = s^2t$ and $y = s + 1/t$. Use the chain rule to express the partial derivatives w_s and w_t in terms of w_x , w_y , s , and t .

7. (10 points) Suppose $z(x, y)$ is a differentiable function satisfying the equation

$$x^2 - y^2 + z^2 - 2z = 4.$$

Find $\partial z/\partial x$ and $\partial^2 z/\partial x^2$ in terms of x, y, z .

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8. Let $A(x, y)$ be the area of the triangle with vertices at the points $P = (0, 0)$, $Q = (1, 2)$, and $R = (x, y)$.
- (a) (5 points) Derive a formula for $A(x, y)$.

- (b) (5 points) Identify all critical points of $A(x, y)$, and classify them as local minima, maxima, or saddle points, with justification.

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(c) (6 points) Use Lagrange multipliers to find a point $R = (x, y)$ on the curve

$$xy = -1$$

which minimizes $A(x, y)$, and determine the minimum area. (hint: it might be easier to minimize A^2)