Math 53 HW 8
Spring 2018

15.7

4 a)

Since $(x,y,z) = (-\sqrt{2}, \sqrt{2}, 1)$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{(-\sqrt{2})^2+(\sqrt{2})^2+1^2} = \sqrt{2+2+1} = \sqrt{5}$$
$$\theta = \arctan(\frac{y}{x}) = \arctan(\frac{\sqrt{2}}{-\sqrt{2}}) = -\frac{\pi}{4}$$
$$z = z = 1$$

b)

Since $(x,y,z) = (2,2,2)$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{2^2+2^2+2^2} = \sqrt{4+4+4} = 2\sqrt{3}$$
$$\theta = \arctan(\frac{y}{x}) = \arctan(\frac{2}{2}) = \frac{\pi}{4}$$
$$z = z = 2$$

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$$r = 2\sin(\theta)$$
$$r^2 = 2r\sin(\theta)$$
$$x^2 + y^2 = 2y$$
$$x^2 + (y-1)^2 = 1$$

The surface is a cylinder of radius 1 centered along the $(0,1,z)$ axis.

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See diagram attached.

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See diagram attached.

$$\int\int\int_E (x-y)dV = \int\int_R \int_0^{y+4} (x-y)dzdA$$
$$= \int\int_R (x-y)(y+4)dA = \int\int_R xy - y^2 + 4x - 4ydA$$
$$= \int_0^{2\pi}\int_1^4 (r^2\sin\theta\cos\theta - r^2\sin^2\theta + 4r\cos\theta - 4r\sin\theta)rdrd\theta$$

$$= \int_0^{2\pi} [\frac{r^4}{4}]_1^4 \sin\theta\cos\theta - [\frac{r^4}{4}]_1^4 \sin^2\theta + [2r^2]_1^4 \cos\theta - [2r^2]_1^4 \sin\theta d\theta$$
$$= \int_0^{2\pi} 63.75\sin\theta\cos\theta - 63.75\sin^2\theta + 30\cos\theta - 30\sin\theta d\theta$$
$$= 63.75(0) - 63.75(\pi) + 30(0) - 30(0) = 63.75\pi$$

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Density $= r$

E is a sphere of radius $r = a$ centered at the origin

$$\text{Mass} = \int\int\int_E r dV = \int_0^{2\pi} \int_0^a \int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} r^2 dz dr d\theta$$
$$= 2\pi \int_0^a 2r^2\sqrt{a^2-r^2} dr = 4\pi \int_0^a r^2\sqrt{a^2-r^2} dr$$

Using substitution $r = a\sin\phi$.

$$= 4\pi \int_0^{\pi/2} (a^2\sin^2\phi)(a\cos\phi)(a\cos\phi) d\phi$$
$$= 4\pi a^4 \int_0^{\pi/2} \sin^2\phi\cos^2\phi d\phi = \pi a^4 \int_0^{\pi/2} \sin^2(2\phi) d\phi = \pi a^4 \int_0^{\pi/2} \frac{1-\cos(4\phi)}{2} d\phi = \frac{\pi^2 a^4}{4}$$

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$$\int_{-3}^{3} \int_0^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} \sqrt{x^2+y^2} dz dy dx = \int_0^{\pi} \int_0^3 \int_0^{9-r^2} r^2 dz dr d\theta$$
$$= \pi \int_0^3 9r^2 - r^4 dr = \pi[3r^3 - \frac{r^5}{5}]_0^3 = 32.4\pi$$

15.8

4 a)

Since $(x, y, z) = (1, 0, \sqrt{3})$

$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{1^2+0^2+\sqrt{3}^2} = 2$$
$$\phi = \arctan(\frac{\sqrt{x^2+y^2}}{z}) = \arctan(\frac{\sqrt{1^2+0^2}}{\sqrt{3}}) = \arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$
$$\theta = \arctan(\frac{y}{x}) = \arctan(\frac{0}{1}) = 0$$

b)

Since $(x, y, z) = (\sqrt{3}, -1, 2\sqrt{3})$

$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{\sqrt{3}^2 + (-1)^2 + (2\sqrt{3})^2} = \sqrt{3+1+12} = 4$$

$$\phi = \arctan(\frac{\sqrt{x^2+y^2}}{z}) = \arctan(\frac{\sqrt{\sqrt{3}^2+(-1)^2}}{2\sqrt{3}}) = \arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$\theta = \arctan(\frac{y}{x}) = \arctan(\frac{-1}{\sqrt{3}}) = -\frac{\pi}{6}$$

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$$\rho = \cos(\phi)$$
$$\rho^2 = \rho\cos(\phi)$$
$$x^2+y^2+z^2 = z$$
$$x^2+y^2+(z-\frac{1}{2})^2 = \frac{1}{4}$$

The surface is a sphere of radius $\frac{1}{2}$ centered at $(0,0,\frac{1}{2})$ axis.

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See diagram attached.

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$$\int\int\int_E f(x,y,z)dV = \int_{\pi/2}^{2\pi}\int_0^{\pi/2}\int_1^2 f(\rho,\phi,\theta)\rho^2\sin\phi d\rho d\phi d\theta$$

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See diagram attached.

$$\int\int\int_E \sqrt{x^2+y^2+z^2}dV = \int_0^{2\pi}\int_0^{\pi/4}\int_1^2 \rho(\rho^2\sin\phi)d\rho d\phi d\theta$$

$$= 2\pi\int_0^{\pi/4}[\frac{\rho^3}{3}]_1^2\sin\phi d\phi = \frac{14\pi}{3}\int^{\pi/4}\sin\phi d\phi = \frac{14\pi}{3}[-\cos\phi]_0^{\pi/4} = \frac{14\pi}{3}(1-\frac{1}{\sqrt{2}})$$

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Distance from a point inside a ball of radius $a=\rho$.

$$\text{Avg. Distance} = \int_0^{2\pi}\int_0^{\pi}\int_0^a \rho(\rho^2\sin\phi)d\rho d\phi d\theta = 2\pi\int_0^{\pi}[\frac{\rho^4}{4}]_0^a\sin\phi d\phi$$

$$= \frac{\pi a^4}{2}\int_0^{\pi}\sin\phi d\phi = \pi a^4$$

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See diagram attached.

$$\text{Volume} = \int_0^{\pi/6}\int_0^{\pi}\int_0^{a} \rho^2 \sin\phi \, d\rho d\phi d\theta = \frac{\pi}{6}\int_0^{\pi} \frac{a^3}{3}\sin\phi d\phi = \frac{\pi a^3}{18}(2) = \frac{\pi a^3}{9}$$

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$$\int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2z + y^2z + z^3) dzdxdy$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} (\rho)(\rho^2\cos\phi)(\rho^2\sin\phi) d\rho d\phi d\theta = 2\pi\int_0^{\pi} \frac{a^6}{6}\sin\phi\cos\phi d\phi$$

$$= \frac{\pi a^6}{6}\int_0^{\pi} \sin(2\phi) d\phi = 0$$