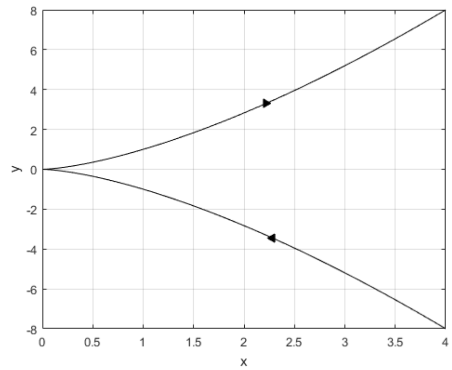
Math 53 HW 2
Spring 2018

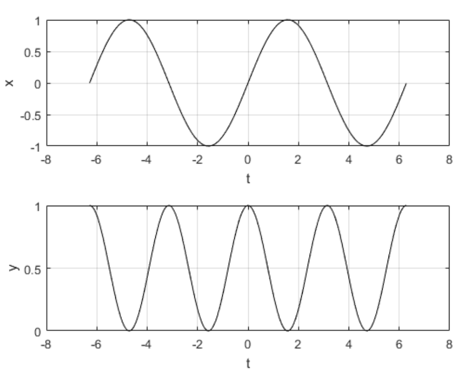
10.1

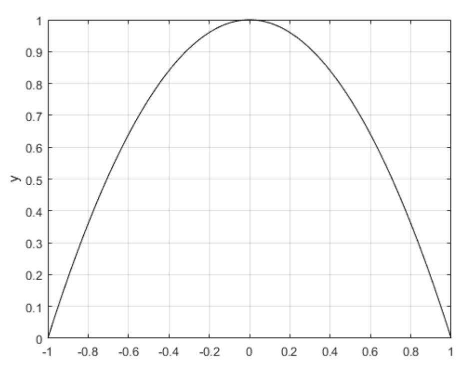
10 a) Parametric plot of $x = t^2, y = t^3$ is given below.



b) Let $\mathbf{r} =< f(t), g(t) >$. If $f(t)$ or $g(t)$ is invertible, we can eliminate the parameter. In this case, $y = t^3$ is invertible.
$\therefore t = \sqrt[3]{y}$
$x = t^2 = y^{\frac{2}{3}}$

22 The particle oscillate along the x direction from $x = -1$ to $x = 1$ every 2π s and along the y direction from $y = 0$ to $y = 1$ every π s.
Combining the motion along the x and y direction, the particle oscillates along the parabola $y = 1 - x^2$ from the point $(-1, 0)$ to $(1, 0)$ every 2π s.

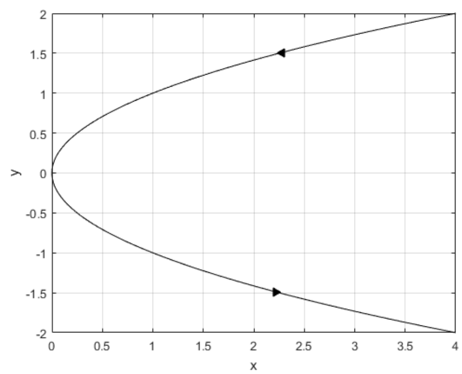




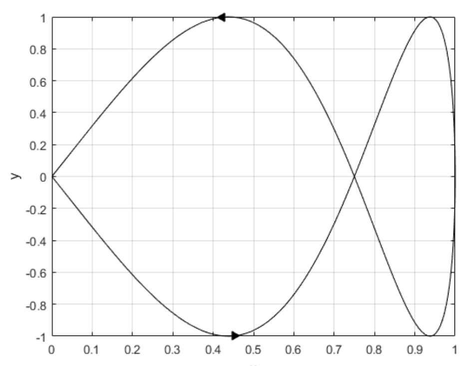
24

x vs t and y vs t	y vs x	Reason
a	III	Parametric curve starts from $(1,0)$ and ends at $(1,0)$
b	I	Parametric curve starts from $(0,0)$ and ends at $(0,0)$
c	IV	Parametric curve starts from $(-2,0)$ and ends at $(2,0)$
d	II	Parametric curve starts from $(2,-2)$ and ends at $(2,-2)$

25 Parametric plot is given below.



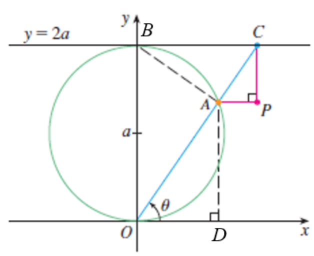
26 Parametric plot is given below.



28

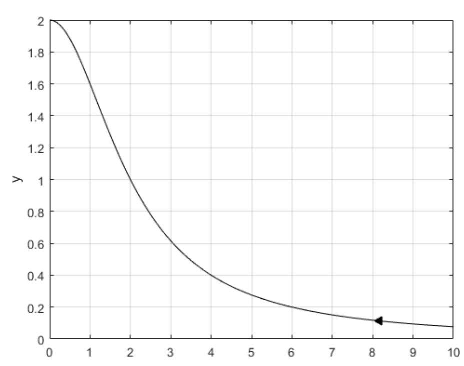
x vs t and y vs t	y vs x	Reason
a	V	(x, y) asymptotically goes to (∞, ∞) as $t \to -\infty$ and $t \to \infty$
b	I	(x, y) asymptotically goes to (∞, ∞) as $t \to \infty$, Domain is restricted to $t \geq 0$
c	II	Parametric equation must formed a closed loop as $\mathbf{r}(0) = \mathbf{r}(2n\pi)$, Since $y = \sin(t)\cos(x) + \cos(t)\sin(x)$, smaller loops must be formed around the region $x = \pm 1$
d	VI	Parametric equation must formed a closed loop as $\mathbf{r}(0) = \mathbf{r}(2n\pi)$
e	IV	(x, y) asymptotically goes to (∞, ∞) as $t \to \infty$ and $(-\infty, \infty)$ as $t \to -\infty$, Parametric equation has oscillatory behavior
f	III	Parametric equation must formed a closed loop as $\mathbf{r}(0) = \mathbf{r}(2n\pi)$, Length of loops (i.e. $n = 1, 2, ...$) asymptotically approaches 0

43 Diagram used for derivation of formula is given below.



1. Let us find the x - coordinates x_C for point C. Using the right angle triangle $\triangle OCB$, $\tan(\theta) = \frac{2a}{x_C}$. Therefore, $x_C = 2a\cot(\theta)$
2. Let us now find the y - coordinates y_A for point A. Using the right angle triangle $\triangle OAB$, $|\mathbf{OA}| = 2a\sin(\theta)$. Therefore, using the right angle triangle $\triangle OAD$ we can show that $y_A = |\mathbf{OA}|\sin(\theta) = 2a\sin^2(\theta)$.
3. Coordinates of point P $(x_P, y_P) = (x_C, y_A) = (2a\cot(\theta), 2a\sin^2(\theta))$

The plot of the parametric equation is given below.

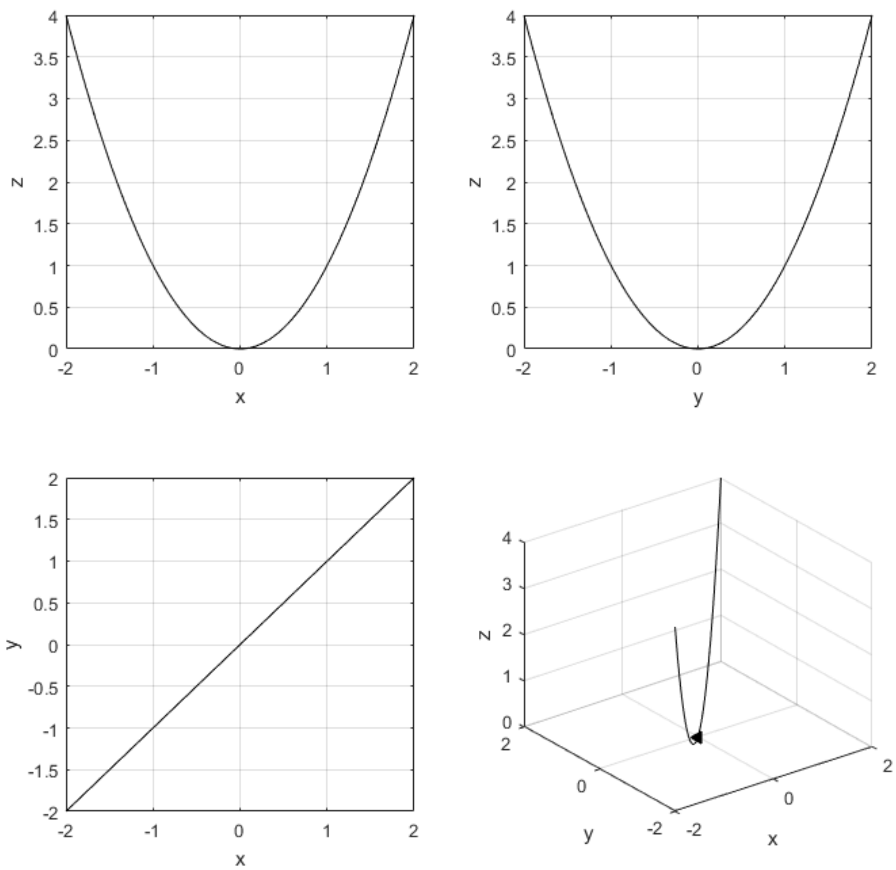


13.1

4

$$\lim_{t\to 1}\left(\frac{t^2-1}{t-1}\mathbf{i}+\sqrt{t+8}\mathbf{j}+\frac{\sin(\pi t)}{\ln(t)}\mathbf{k}\right)=\left(\lim_{t\to 1}\frac{t^2-1}{t-1}\mathbf{i}+\lim_{t\to 1}\sqrt{t+8}\mathbf{j}+\lim_{t\to 1}\frac{\sin(\pi t)}{\ln(t)}\mathbf{k}\right)$$
$$=\left(\lim_{t\to 1}\frac{(t-1)(t+1)}{t-1}\mathbf{i}+\lim_{t\to 1}\sqrt{t+8}\mathbf{j}+\lim_{t\to 1}\frac{\pi\cos(\pi t)}{(\frac{1}{t})}\mathbf{k}\right)$$
$$=\left(\lim_{t\to 1}(t+1)\mathbf{i}+\lim_{t\to 1}\sqrt{t+8}\mathbf{j}+\lim_{t\to 1}\pi t\cos(\pi t)\mathbf{k}\right)=(2\mathbf{i}+9\mathbf{j}-\pi\mathbf{k})$$

16 The plot of the projections and the parametric equation are given below.



21 II
Reasons:

1. The projection of the parametric curve on the xz plane will be a spiral with linearly increasing radius.
2. The y coordinate of the parametric curve must increase linearly.

22 VI
Reasons:

1. The projection of the parametric curve on the xy plane will be a circle of radius 1.
2. The parametric curve reaches $z = 1$ for $t = 0$.
3. The z coordinate of the parametric curve asymptotically approaches 0 for $t \to \pm\infty$.

23 V
Reasons:

1. The parametric curve touches the xy plane at $(0, 1, 0)$ at $t = 0$.
2. The parametric curve approaches $(\pm\infty, 0, \infty)$ as $t \to \pm\infty$.

24 I
Reasons:

1. The projection of the parametric curve on the xy plane will be a circle of radius 1.
2. The parametric curve oscillates in the z direction from -1 to 1.

25 IV
Reasons:

1. The projection of the parametric curve on the xy plane will be a circle of radius 1.
2. The z coordinate of the parametric curve exponentially increases to ∞ as t increases.

26 III
Reasons:

1. The parametric curve must exist in the $x + y = 1$ plane.
2. The projection of the parametric curve on the xz plane and the yz plane corresponds to oscillations from $x = 0$ to $x = 1$ and from $y = 0$ to $y = 1$ respectively.

28

1. Let us look at the projection along the xy plane.

   $$x^2 + y^2 = sin^2(t) + cos^2(t) = 1$$

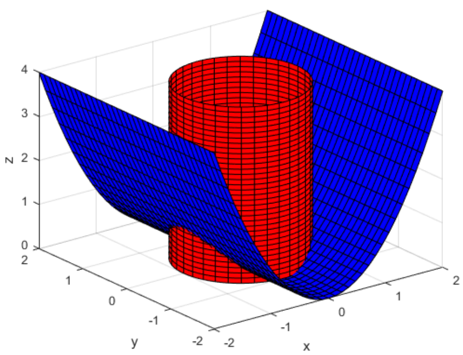
   Therefore, the parametric equation must lie on the cylinder $x^2 + y^2 = 1$.

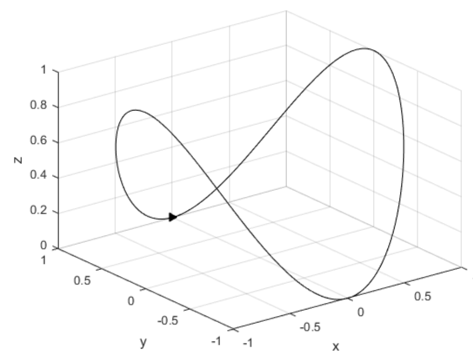
2. Let us look at the projection along the xz plane.

$$x^2 = sin^2(t) = z$$

Therefore, the parametric equation must lie on the parabolic sheet $z = x^2$.

The plot of the two surfaces and the parametric curve is given below.





32 Substituting the parametric equation $\mathbf{r}(t) =< \sin(t), \cos(t), t >$ into the equation of the sphere $x^2+y^2+z^2 = 5$ we get the following.

$$sin^2(t) + cos^2(t) + t^2 = 5$$
$$1 + t^2 = 5$$
$$t^2 = 4$$
$$t = \pm 2$$

Therefore the two points of intersection are $\mathbf{r}(-2) =< -\sin(2), \cos(2), -2 >$ and $\mathbf{r}(2) =< \sin(2), \cos(2), 2 >$.
41

1. Let us test if point $(1, 4, 0)$ passes through the parametric equation $\mathbf{r}(t) =< t^2, 1 - 3t, 1 + t^3 >$.

$$x : t^2 = 1 \implies t = \pm 1$$

$$y : 1 - 3(\pm 1) = -2 \text{ or } 4 \implies t = -1$$
$$z : 1 + (-1)^3 = 1 - 1 = 0$$

Therefore, the point passes through the parametric equation.

2. Let us test if point $(9, -8, 28)$ passes through the parametric equation $\mathbf{r}(t) =< t^2, 1 - 3t, 1 + t^3 >$.

$$x : t^2 = 9 \implies t = \pm 3$$

$$y : 1 - 3(\pm 3) = -8 \text{ or } 10 \implies t = 3$$
$$z : 1 + (3)^3 = 1 + 27 = 28$$

Therefore, the point passes through the parametric equation.

3. Let us test if point $(4, 7, -6)$ passes through the parametric equation $\mathbf{r}(t) =< t^2, 1-3t, 1+t^3 >$.

$$x : t^2 = 4 \implies t = \pm 2$$

$$y : 1 - 3(\pm 2) = -5 \text{ or } 7 \implies t = -2$$

$$z : 1 + (-2)^3 = 1 - 8 = -7 \neq -6$$

Therefore, the point does not pass through the parametric equation.

42

1. Since the parametric curve is on the cylinder $x^2 + y^2 = 4$, the parameterization of x and y coordinate is as follows.

$$x = 2\cos(t)$$

$$y = 2\sin(t)$$

2. Since the parametric curve is on the surface $z = xy$, the parameterization of z coordinate is as follows.

$$z = 2\cos(t)2\sin(t) = 4\sin(t)\cos(t) = 2\sin(2t)$$

Therefore, the parametric equation is $\mathbf{r}(t) =< 2\cos(t), 2\sin(t), 2\sin(2t) >$.

50 In order to find the intersection between $\mathbf{r}_1(t) =< t, t^2, t^3 >$ and $\mathbf{r}_2(s) =< 1+2s, 1+6s, 1+14s >$, we need to solve the following simultaneous equations.

$$x : t = 1 + 2s$$

$$y : t^2 = 1 + 6s \implies (1+2s)^2 = 1 + 6s \implies 1 + 4s + 4s^2 = 1 + 6s \implies 4s^2 - 2s = 0 \implies s = 0 \text{ or } \frac{1}{2}$$

Therefore, $t = 1$ or 2.

$$z : t^3 = 1 + 14s \implies 1^3 = 1 + 14(0) \text{ or } 2^3 = 1 + 14(\frac{1}{2})$$

Therefore, the two parametric curves intersect at the points $\mathbf{r}_1(2) = \mathbf{r}_2(\frac{1}{2}) =< 2, 4, 8 >$ and $\mathbf{r}_1(1) = \mathbf{r}_2(0\frac{1}{2}) =< 1, 1, 1 >$. However, the particles do not collide as they do not reach the point at the same time.

53 The proofs of the limit theorem are given below.

a)

$$\lim_{t\to a}[\mathbf{u}(t) + \mathbf{v}(t)] = \lim_{t\to a}[< x_u(t), y_u(t), z_u(t) > + < x_v(t), y_v(t), z_v(t) >]$$

$$= \lim_{t\to a} < x_u(t)+x_v(t), y_u(t)+y_v(t), z_u(t)+z_v(t) >=< \lim_{t\to a}[x_u(t)+x_v(t)], \lim_{t\to a}[y_u(t)+y_v(t)], \lim_{t\to a}[z_u(t)+z_v(t)] >$$

$$=< \lim_{t\to a} x_u(t) + \lim_{t\to a} x_v(t), \lim_{t\to a} y_u(t) + \lim_{t\to a} y_v(t), \lim_{t\to a} z_u(t) + \lim_{t\to a} z_v(t)] >$$

$$= \lim_{t\to a} < x_u(t), y_u(t), z_u(t) > + \lim_{t\to a} < x_v(t), y_v(t), z_v(t) >= \lim_{t\to a} \mathbf{u}(t) + \lim_{t\to a} \mathbf{v}(t)$$

b)

$$\lim_{t\to a}[c\mathbf{u}(t)] = \lim_{t\to a} < cx_u(t), cy_u(t), cz_u(t) >=< \lim_{t\to a}[cx_u(t)], \lim_{t\to a}[cy_u(t)], \lim_{t\to a}[cz_u(t)] >$$

$$=< c\lim_{t\to a} x_u(t)], c\lim_{t\to a} y_u(t), c\lim_{t\to a} z_u(t) >= c\lim_{t\to a} \mathbf{u}(t)$$

c)

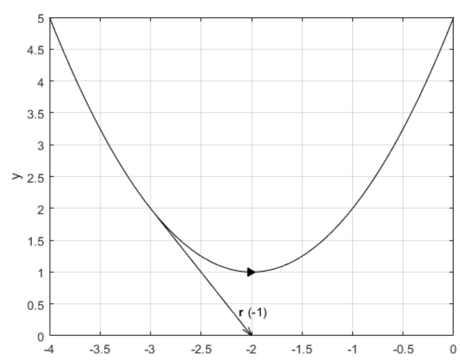
$$\lim_{t\to a}[\mathbf{u}(t)\cdot\mathbf{v}(t)] = \lim_{t\to a}[< x_u(t), y_u(t), z_u(t) > \cdot < x_v(t), y_v(t), z_v(t) >]$$
$$= \lim_{t\to a}[x_u(t)x_v(t) + y_u(t)y_v(t) + z_u(t)z_v(t)] = \lim_{t\to a}[x_u(t)x_v(t)] + \lim_{t\to a}[y_u(t)y_v(t)] + \lim_{t\to a}[z_u(t)z_v(t)]$$
$$= \lim_{t\to a} x_u(t)\lim_{t\to a} x_v(t) + \lim_{t\to a} y_u(t)\lim_{t\to a} y_v(t) + \lim_{t\to a} z_u(t)\lim_{t\to a} z_v(t)$$
$$=< \lim_{t\to a} x_u(t), \lim_{t\to a} y_u(t), \lim_{t\to a} z_u(t) > \cdot < \lim_{t\to a} x_v(t), \lim_{t\to a} y_v(t), \lim_{t\to a} z_v(t) >$$
$$= \lim_{t\to a} < x_u(t), y_u(t), z_u(t) > \cdot \lim_{t\to a} < x_v(t), y_v(t), z_v(t) >= \lim_{t\to a}\mathbf{u}(t)\cdot\lim_{t\to a}\mathbf{v}(t)$$

d)

$$\lim_{t\to a}[\mathbf{u}(t)\times\mathbf{v}(t)] = \lim_{t\to a}[< x_u(t), y_u(t), z_u(t) > \times < x_v(t), y_v(t), z_v(t) >]$$
$$= \lim_{t\to a} < y_u(t)z_v(t) - y_v(t)z_u(t), z_u(t)x_v(t) - z_v(t)x_u(t), x_u(t)y_v(t) - x_v(t)y_u(t) >$$
$$=< \lim_{t\to a} y_u(t)\lim_{t\to a} z_v(t) - \lim_{t\to a} y_v(t)\lim_{t\to a} z_u(t), \lim_{t\to a} z_u(t)\lim_{t\to a} x_v(t) - \lim_{t\to a} z_v(t)\lim_{t\to a} x_u(t), ...$$
$$... \lim_{t\to a} x_u(t)\lim_{t\to a} y_v(t) - \lim_{t\to a} x_v(t)\lim_{t\to a} y_u(t) >$$
$$= \lim_{t\to a}\mathbf{u}(t)\times\lim_{t\to a}\mathbf{v}(t)$$

13.2

3 a) and c) The parametric equation $\mathbf{r}(t) =< t-2, t^2+10 >$ and the vector $\mathbf{r}'(-1)$



b) $\mathbf{r}'(t) =< 1, 2t >$

19 The derivative of the $\mathbf{r}(t) =< \cos(t), 3t, 2\sin(2t) >$ at $t = 0$ is found below.

$$\mathbf{r}'(t) =< -\sin(t), 3, 4\cos(2t) >$$
$$\mathbf{r}'(0) =< -\sin(0), 3, 4\cos(0) >=< 0, 3, 4 >$$

Therefore the tangent vector $\mathbf{T}$ is calculated below.

$$\mathbf{T}(0) = \frac{\mathbf{r}'(0)}{|\mathbf{r}'(0)|} = \frac{< 0,3,4 >}{| < 0,3,4 > |} = \frac{1}{\sqrt{0+9+16}} < 0,3,4 >= \frac{1}{5} < 0,3,4 >$$

27 The parametric curve formed by the intersection of the two cylinders is derived below.

1. Since the parametric curve is on the cylinder $x^2 + y^2 = 25$, the parameterization of x and y coordinate is as follows.

$$x = 5\cos(t)$$
$$y = 5\sin(t)$$

2. Since the parametric curve is on the cylinder $y^2 + z^2 = 20$, the parameterization of z coordinate is as follows.

$$z = \pm\sqrt{20 - y^2} = \pm\sqrt{20 - 25\sin^2(t)}$$

Therefore, the $\mathbf{r}(t) =< 5\cos(t), 5\sin(t), \pm\sqrt{20 - 25\sin^2(t)} >$. The point $(3, 4, 2)$ takes place at $t = \arccos(\frac{3}{5})$. The derivative of the $\mathbf{r}'(t)$ at $t = \arccos(\frac{3}{5})$ is found below.

$$\mathbf{r}'(t) =< -5\sin(t), 5\cos(t), \frac{-25\sin(t)\cos(t)}{\sqrt{20 - 25\sin^2(t)}} >$$

$$\mathbf{r}'(\arccos(\frac{3}{5})) =< -5(\frac{4}{5}), 5(\frac{3}{5}), \frac{-25(\frac{4}{5})(\frac{3}{5})}{\sqrt{20 - 25(\frac{4}{5})^2}} >=< -4, 3, -6 >$$

The parametric equation of the tangent line is given below.

$$\mathbf{r}_{\text{tangent}}(s) =< 3, 4, 2 > +s < -4, 3, -6 >$$

28 The derivative of the parametric curve $\mathbf{r}(t) =< 2\cos(t), 2\sin(t), e^t >$ is given below.

$$\mathbf{r}'(t) =< -2\sin(t), 2\cos(t), e^t >$$

Since the tangent line of the parametric curve is parallel to the plane $\sqrt{3}x + y = 1$, the direction vector of the tangent line $\mathbf{r}'(t)$ must be orthogonal to the normal vector $\mathbf{n} =< \sqrt{3}, 1, 0 >$ of the plane.

$$\therefore < -2\sin(t), 2\cos(t), e^t > \cdot < \sqrt{3}, 1, 0 >= 0$$
$$-2\sqrt{3}\sin(t) + 2\cos(t) = 0$$
$$\cos(t) = \sqrt{3}\sin(t)$$
$$\tan(t) = \frac{1}{\sqrt{3}}$$
$$t = \frac{\pi}{6}$$

Therefore, the point on the curve is $\mathbf{r}(\frac{\pi}{6}) =< 2\cos(\frac{\pi}{6}), 2\sin(\frac{\pi}{6}), e^{\frac{\pi}{6}} >=< \sqrt{3}, 1, e^{\frac{\pi}{6}} >$.

33 The derivative of the $\mathbf{r}_1(t) =< t, t^2, t^3 >$ and $\mathbf{r}_2(s) =< \sin(s), \sin(2s), s >$ at $t = 0$ and $s = 0$ is given below.

1.

$$\mathbf{r}_1'(t) =< 1, 2t, 3t^2 >$$
$$\mathbf{r}_1'(0) =< 1, 0, 0 >$$

2.

$$\mathbf{r}_2'(t) =< \cos(s), 2\cos(2s), 1 >$$
$$\mathbf{r}_2'(0) =< 1, 2, 1 >$$

Therefore the angle between the $\mathbf{r}_1'(0)$ and $\mathbf{r}_2'(0)$ is found below.

$$\cos(\theta) = \frac{< 1,0,0 > \cdot < 1,2,1 >}{| < 1,0,0 > || < 1,2,1 > |} = \frac{1+0+0}{(1)(\sqrt{6})} = \frac{1}{\sqrt{6}}$$
$$\theta = 1.15$$

44

$$\frac{d}{dt}[f(t)\mathbf{u}(t)] = \frac{d}{dt}[< f(t)x_u(t), f(t)y_u(t), f(t)z_u(t) >]$$
$$=< \frac{d}{dt}[f(t)x_u(t)], \frac{d}{dt}[f(t)y_u(t)], \frac{d}{dt}[f(t)z_u(t)] >$$
$$=< f'(t)x_u(t) + f(t)x_u'(t), f(t)'y_u(t) + f(t)y_u'(t), f(t)'z_u(t) + f(t)z_u'(t) >$$
$$=< f'(t)x_u(t), f(t)'y_u(t), f(t)'z_u(t) > + < f(t)x_u'(t), f(t)y_u'(t), f(t)z_u'(t) >$$
$$= f'(t) < x_u(t), y_u(t), z_u(t) > + f(t) < x_u'(t), y_u'(t), z_u'(t) > = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)$$

45

$$\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \frac{d}{dt}[< x_u(t), y_u(t), z_u(t) > \times < x_v(t), y_v(t), z_v(t) >]$$
$$= \frac{d}{dt} < y_u(t)z_v(t) - y_v(t)z_u(t), z_u(t)x_v(t) - z_v(t)x_u(t), x_u(t)y_v(t) - x_v(t)y_u(t) >$$
$$=< \frac{d}{dt}[y_u(t)z_v(t) - y_v(t)z_u(t)], \frac{d}{dt}[z_u(t)x_v(t) - z_v(t)x_u(t)], \frac{d}{dt}[x_u(t)y_v(t) - x_v(t)y_u(t)] >$$
$$=< y_u'(t)z_v(t) - y_v(t)z_u'(t), z_u'(t)x_v(t) - z_v(t)x_u'(t), x_u'(t)y_v(t) - x_v(t)y_u'(t) > + ...$$
$$< y_u(t)z_v'(t) - y_v'(t)z_u(t), z_u(t)x_v'(t) - z_v'(t)x_u(t), x_u(t)y_v'(t) - x_v'(t)y_u(t) >$$
$$= \mathbf{u}'(t) \times \mathbf{v}'(t)$$

56 If the $\mathbf{r}(t)$ is perpendicular to $\mathbf{r}'(t)$, the we get the following equation.

$$\mathbf{r}'(t) \cdot \mathbf{r}(t) = 0 \implies 2\mathbf{r}'(t) \cdot \mathbf{r}(t) = 0 \implies \frac{d}{dt}[\mathbf{r}(t) \cdot \mathbf{r}(t)] = 0 \implies \mathbf{r}(t) \cdot \mathbf{r}(t) = k$$

Where $k \in \mathbb{R}^+$.
The surface $\mathbf{r}(t) \cdot \mathbf{r}(t) = x^2 + y^2 + z^2 = k$ is a sphere centered at the origin with radius $\sqrt{k}$.

13.3

5 The ds for $\mathbf{r}(t) =< 1, t^2, t^3 >$ is calculated below.

$$ds = |\mathbf{r}'(t)|dt = | < 0, 2t, 3t^2 > |dt = \sqrt{0^2 + (2t)^2 + (3t^2)^2}dt = \sqrt{4t^2 + 9t^4}dt = |t|\sqrt{4 + 9t^2}dt$$

We can drop the absolute value as $0 \leq t \leq 1$. Therefore, the length of the parametric curve is calculated below.

$$s = \int_0^1 t\sqrt{4 + 9t^2}dt = \int_4^{13} t\sqrt{u}\frac{du}{18t} = \frac{1}{18}\int_4^{13} \sqrt{u}du = \frac{1}{18}[\frac{2}{3}u^{\frac{3}{2}}]_4^{13} = \frac{\sqrt{2197} - 8}{27}$$

11 The parametric curve formed by the intersection of the two surfaces is derived below.

1. Since the parametric curve is on the parabolic cylinder $x^2 = 2y$, the parameterization of x and y coordinate is as follows.
$$x = t$$
$$y = \frac{t^2}{2}$$

2. Since the parametric curve is on the cylinder $3z = xy$, the parameterization of z coordinate is as follows.
$$z = \frac{xy}{3} = \frac{t\frac{t^2}{2}}{3} = \frac{t^3}{6}$$

The ds for $\mathbf{r}(t) =< t, \frac{t^2}{2}, \frac{t^3}{6} >$ is calculated below.

$$ds = |\mathbf{r}'(t)|dt = | < 1, t, \frac{t^2}{2} > | = \sqrt{1^2 + t^2 + \left(\frac{t^2}{2}\right)^2} dt = \sqrt{\left(\frac{t^2}{2} + 1\right)^2} dt = \left(\frac{t^2}{2} + 1\right) dt$$

The length of the parametric curve from the origin $\mathbf{r}(0) =< 0, 0, 0 >$ to the point $\mathbf{r}(6) =< 6, \frac{6^2}{2}, \frac{6^3}{6} >=< 6, 18, 36 >$ is calculated below.

$$s = \int_0^6 \left(\frac{t^2}{2} + 1\right) dt = \left[\frac{t^3}{6} + t\right]_0^6 = 36 + 6 - 0 - 0 = 42$$

16 The ds for $\mathbf{r}(t) =< \frac{2}{t^2+1} - 1, \frac{2t}{t^2+1} >$ is calculated below.

$$ds = |\mathbf{r}'(t)|dt = | < -\frac{4t}{(t^2+1)^2}, \frac{2}{t^2+1} - \frac{4t^2}{(t^2+1)^2} > |dt = |\frac{2}{(t^2+1)^2} < -2t, t^2 + 1 - 2t^2 > |dt$$

$$= \frac{2}{(t^2+1)^2}| < -2t, -t^2 + 1 > |dt = \frac{2}{(t^2+1)^2}\sqrt{4t^2 + t^4 + 1 - 2t^2}dt = \frac{2}{(t^2+1)^2}\sqrt{t^4 + 1 + 2t^2}dt$$
$$= \frac{2}{(t^2+1)^2}(t^2+1)dt = \frac{2}{t^2+1}dt$$

The length of the parametric curve from the point $\mathbf{r}(0) =< 0, 1 >$ to the point $\mathbf{r}(t) =< \frac{2}{t^2+1} - 1, \frac{2t}{t^2+1} >$ is calculated below.
$$s = \int_0^t \frac{2}{t^2+1}dt = 2[\arctan t]_0^t = 2\arctan(t)$$

Therefore, we get the relationship $t = \tan(\frac{s}{2})$. Plugging in the relationship $t = \tan(s)$ into the $\mathbf{r}(t)$, we get the following.

$$\mathbf{r}(s) =< \frac{2}{\tan^2(\frac{s}{2}) + 1} - 1, \frac{2\tan\frac{s}{2})}{\tan^2(\frac{s}{2}) + 1} >=< \frac{2}{\sec^2(\frac{s}{2})} - 1, \frac{2\tan(\frac{s}{2})}{\sec^2(\frac{s}{2})} >=< 2\cos^2(\frac{s}{2}) - 1, 2\tan(\frac{s}{2})\cos^2(\frac{s}{2}) >$$

$$=< 2\cos^2(\frac{s}{2}) - 1, 2\sin(\frac{s}{2})\cos(\frac{s}{2}) >=< \cos(s), \sin(s) >$$

The parametric equation $\mathbf{r}(s) =< \cos(s), \sin(s) >$ corresponds to a circle of radius 1 centered at the origin.