3. By Stokes’ theorem, we have
\[ \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}, \]
where \( C \) is the boundary circle \( x^2 + z^2 = 16, y = 0 \), oriented appropriately. We can parametrize this curve via \( \mathbf{r}(t) = \langle 4 \cos t, 0, -4 \sin t \rangle \), so
\[ \int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} \langle -4 \sin t, 4 \cos t, 0 \rangle \cdot \langle -4 \sin t, 0, -4 \cos t \rangle \ dt = \int_0^{2\pi} 16 \sin^2 t \ dt = 16\pi. \]

5. Let \( R \) be the square with vertices \((\pm 1, \pm 1, -1)\) oriented upward, so that \( S \cup R \) is the cube with vertices \((\pm 1, \pm 1, \pm 1)\). Then by Stokes’ theorem,
\[ \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} = \iint_R \text{curl} \mathbf{F} \cdot d\mathbf{S} \]
since they share the same boundary. Now \( \text{curl} \mathbf{F} = \langle x^2, xy - 2xyz, y - x \rangle \), and the unit normal to \( R \) is just \( \langle 0, 0, 1 \rangle \), so we have
\[ \iint_R \text{curl} \mathbf{F} \cdot d\mathbf{S} = \int_{-1}^1 \int_{-1}^1 \langle -x^2, xy + 2xyz, y + x \rangle \cdot \langle 0, 0, 1 \rangle \ dx \ dy = \int_{-1}^1 \int_{-1}^1 y + x \ dx \ dy = 0. \]

9. By Stokes’ theorem, we have
\[ \int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S}, \]
where \( S \) is the portion of the paraboloid in the first octant, oriented upward. The surface is parametrized by \( x \) and \( y \) satisfying \( x, y \geq 0 \) and \( x^2 + y^2 \leq 1 \), and the normal at \((x, y, z)\) is just the negative of the gradient, namely, \( \langle 2x, 2y, 1 \rangle \). Now \( \text{curl} \mathbf{F} = (-y, -z, -x) \),
and so
\[
\iint_S \text{curl} \mathbf{F} \cdot dS = \int_0^1 \int_0^{\sqrt{1-x^2}} (-y, x^2 + y^2 - 1, -x) \cdot (2x, 2y, 1) \ dy \ dx
\]
\[
= \int_0^1 \int_0^{\sqrt{1-x^2}} -2xy + 2x^2y + 2y^3 - 2y - x \ dy \ dx
\]
\[
= \int_0^{\pi/2} \int_0^1 2r^3 \cos \theta \sin \theta + 2r^4 \cos^2 \theta \sin \theta + 2r^4 \sin^3 \theta - 2r^2 \sin \theta - r^2 \cos \theta \ dr \ d\theta
\]
\[
= \int_0^{\pi/2} -\frac{1}{2} \cos \theta \sin \theta + \frac{2}{5} \cos^2 \theta \sin \theta + \frac{2}{5} \sin^3 \theta - \frac{2}{3} \sin \theta - \frac{1}{3} \cos \theta \ d\theta.
\]
After writing \( \cos \theta \sin \theta \) as \( \frac{1}{2} \sin 2\theta \) and \( \cos^2 \theta \sin \theta + \sin^3 \theta \) as \( \sin \theta \), this becomes
\[
\int_0^{\pi/2} -\frac{1}{4} \sin 2\theta + \frac{2}{5} \sin \theta - \frac{2}{3} \sin \theta - \frac{1}{3} \cos \theta \ d\theta = -\frac{1}{4} + \frac{2}{3} - \frac{2}{5} - \frac{1}{3} = -\frac{15 + 24 - 20}{60}
\]
\[
= -\frac{51}{60}
\]
\[
= -\frac{17}{20}.
\]

15. Let \( C \) be the boundary of \( S \), with the orientation induced by that of \( S \). We can parametrize \( C \) via \( \mathbf{r}(t) = (\cos t, 0, -\sin t) \), and then
\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} (-\sin t, \cos t) \cdot (-\sin t, 0, -\cos t) \ dt
\]
\[
= \int_0^{2\pi} -\cos^2 t \ dt
\]
\[
= -\pi.
\]

We now evaluate the surface integral over \( S \). The hemisphere can be parametrized as \( \mathbf{r}(x, z) = (x, \sqrt{1 - x^2 - z^2}, z) \), and then the normal is just the negative of the gradient, which is \( \left( \frac{x}{\sqrt{1 - x^2 - z^2}}, -\frac{1}{\sqrt{1 - x^2 - z^2}}, \frac{z}{\sqrt{1 - x^2 - z^2}} \right) \). Combining this with the fact that \( \text{curl} \ \mathbf{F} = (-1, -1, -1) \), this gives
\[
\iint_S \text{curl} \ \mathbf{F} \cdot d\mathbf{S} = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (-1, -1, -1) \cdot \left( \frac{x}{\sqrt{1 - x^2 - z^2}}, -\frac{1}{\sqrt{1 - x^2 - z^2}}, \frac{z}{\sqrt{1 - x^2 - z^2}} \right) \ dz \ dx.
\]
Now since \( x/\sqrt{1 - x^2 - z^2} \) changes signs after replacing \( x \) by \( -x \), the integral will be 0. Similarly, \( z/\sqrt{1 - x^2 - z^2} \) will integrate to 0, so we just get
\[
\iint_S \text{curl} \ \mathbf{F} \cdot d\mathbf{S} = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} -1 \ dz \ dx
\]
\[
= -\pi.
\]
Since this equals the line integral over \( C \), Stokes’ theorem holds in this case.
16. Let $S$ be the surface in the plane $x + y + z = 1$ with boundary $C$. The curl of $\langle z, -2x, 3y \rangle$ is $\langle 3, 1, -2 \rangle$, and the normal to the plane is $\langle 1, 1, 1 \rangle$, so by Stokes’ theorem, we have
\[
\int_C (z \, dx - 2x \, dy + 3y \, dz) = \int_S \langle 3, 1, -2 \rangle \cdot \langle 1, 1, 1 \rangle \, dA
= \int_S 2 \, dA,
\]
which is just twice the surface area of $S$. Thus, the integral depends only on the area enclosed by $C$.

17. To find the work done, we need to compute the line integral over the parallelogram with vertices $(0, 0, 0), (1, 0, 0), (1, 2, 1)$, and $(0, 2, 1)$. By Stokes’ theorem, this is the same as the surface integral of curl $\mathbf{F}$ over the parallelogram. Since the vectors $\langle 1, 0, 0 \rangle$ and $\langle 0, 2, 1 \rangle$ span the parallelogram, we can parametrize it via
\[
r(u, v) = u \langle 1, 0, 0 \rangle + v \langle 0, 2, 1 \rangle = \langle u, 2v, v \rangle,
\]
where $0 \leq u, v \leq 1$. Taking the cross product gives
\[
r_u \times r_v = \langle 0, -1, 2 \rangle.
\]
Finally, curl $\mathbf{F} = \langle 8y, 2z, 2y \rangle$, so putting it all together, we get
\[
\iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S} = \int_0^1 \int_0^1 \langle 16v, 2v, 4v \rangle \cdot \langle 0, -1, 2 \rangle \, du \, dv
= \int_0^1 \int_0^1 6v \, du \, dv
= 3
\]

19. By Stokes’ theorem, the surface integral of curl $\mathbf{F}$ equals the line integral of $\mathbf{F}$ over the boundary of $S$. But $S$ has no boundary, so the line integral over the boundary is just 0.