

16.5

2 a) $\text{curl } F = \langle 4y^3z^3, 2x^3yz, -x^3z^2 \rangle$

b) $\text{div } F = 3yz^2(x^2+y^3)$

7. a) $\text{curl } F = \langle -e^y \cos z, -e^z \cos x, -e^x \cos y \rangle$

b) $\text{div } F = \langle e^x \sin y + e^y \sin z + e^z \sin x \rangle$

9. a) $\text{div } F$ is negative because $\frac{\partial F_y}{\partial y}$ is negative.

b) Yes

10. a) $\text{div } F$ is positive because $\frac{\partial F_x}{\partial x}$ and $\frac{\partial F_y}{\partial y}$ are positive.

b) Yes

11. a) $\text{div } F$ is zero because $\frac{\partial F_x}{\partial x}$ and $\frac{\partial F_y}{\partial y}$ are zero.

b) No, $\text{curl } F$ points downward.

12. a) No, cannot take curl of a scalar field

c) Yes, scalar field

e) No, cannot take grad of vector field

g) Yes, scalar field

i) Yes, vector field

14. $\text{curl } F = \langle 0, -4yxz^3, xz^4 \rangle$, so F is not conservative.

15. $\text{curl } F = \langle -2x \sin y, 0, 2z \sin y \rangle$, so F is not conservative.

19. $\text{div} \langle x \sin y, \cos y, z - xy \rangle = 1$, and thus this cannot be a curl field.

21. $\text{curl } F = \left\langle \frac{\partial F_2}{\partial y} - \frac{\partial F_1}{\partial z}, \frac{\partial F_3}{\partial z} - \frac{\partial F_1}{\partial x}, \frac{\partial F_3}{\partial x} - \frac{\partial F_2}{\partial y} \right\rangle = \langle 0, 0, 0 \rangle$

22. $\text{div } F = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} = 0 + 0 + 0 = 0$.

26. $\text{curl}(fF) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ fF_x & fF_y & fF_z \end{vmatrix} = \left\langle \frac{\partial f}{\partial y} F_z + f \frac{\partial F_z}{\partial y} - \left(\frac{\partial f}{\partial z} F_y + f \frac{\partial F_y}{\partial z} \right), \right.$

$$\left. \frac{\partial f}{\partial z} F_x + f \frac{\partial F_x}{\partial z} - \left(\frac{\partial f}{\partial x} F_z + f \frac{\partial F_z}{\partial x} \right), \right.$$
$$\left. \frac{\partial f}{\partial x} F_y + f \frac{\partial F_y}{\partial x} - \left(\frac{\partial f}{\partial y} F_x + f \frac{\partial F_x}{\partial y} \right) \right\rangle$$

$$= f \text{curl } F + (\nabla f) \times F.$$

16.6

2. a) P does not lie on the surface

b) $u=2, v=1$ gives us Q.

4. paraboloid opening towards the x-axis

6. elliptical cylinder opening along the y-axis

$$22. \quad \begin{aligned} x &= \sin \phi \cos \theta & 0 \leq \theta \leq 2\pi \\ y &= \frac{1}{\sqrt{2}} \cos \phi & 0 \leq \phi \leq \frac{\pi}{2} \\ z &= \frac{1}{\sqrt{3}} \sin \phi \sin \theta \end{aligned}$$

$$24. \quad \begin{aligned} x &= 3 \cos \theta & 0 \leq \theta \leq \pi \\ z &= 3 \sin \theta & -4 \leq y \leq 4 \\ y &= y \end{aligned}$$

$$26. \quad \begin{aligned} x &= x & -1 \leq x \leq 1 \\ y &= y & -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2} \\ z &= x+3 \end{aligned}$$

$$34. \quad \begin{aligned} u=2 \quad v=1 & & r_u = \langle 2u, 0, 1 \rangle \\ r_u \times r_v = \langle -3, -4, 12 \rangle & & r_v = \langle 0, 3v^2, 1 \rangle \end{aligned}$$

$$-3(x-5) - 4(y-2) + 12(z-3) = 0$$

$$36. \quad \begin{aligned} r_u &= \langle \cos u, -\sin u \sin v, 0 \rangle \\ r_v &= \langle 0, \cos u \cos v, \cos v \rangle \end{aligned}$$

$$r_u \times r_v = \left\langle -\frac{\sqrt{3}}{8}, -\frac{3}{4}, \frac{3\sqrt{3}}{8} \right\rangle$$

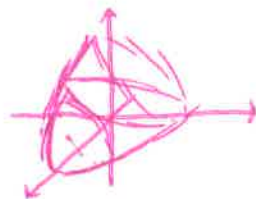
$$-\frac{\sqrt{3}}{8} \left(x - \frac{1}{2}\right) - \frac{3}{4} \left(y - \frac{\sqrt{3}}{4}\right) + \frac{3\sqrt{3}}{8} \left(z - \frac{1}{2}\right) = 0$$

$$37. \quad u=1 \quad v=1$$

$$r_u = \langle -2u, 0, -1 \rangle$$

$$r_v = \langle -2v, -\phi, 0 \rangle$$

$$r_u \times r_v = \langle -1, 2, 2 \rangle$$



$$-1(x+1) + 2(y+1) + 2(z+1) = 0$$