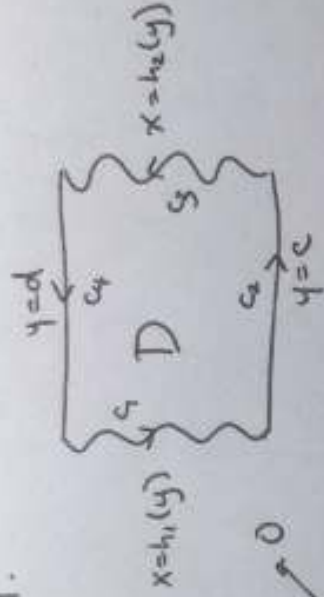


First, we prove (6) for type II regions, or regions

$$D = \{ (x, y) \mid h_1(y) \leq x \leq h_2(y), \quad c \leq y \leq d \}.$$

Then

$$\int_c^d Q \, dy = \int_c^d Q \, dy + \int_{c_2}^d Q \, dy + \int_{c_3}^d Q \, dy + \int_{c_4}^d Q \, dy$$



$$\begin{aligned} &= \int_c^d Q(h_1(y), y) \, dy + \int_c^d Q(h_2(y), y) \, dy \\ &= \int_c^d \int_{h_1(y)}^{h_2(y)} Q_x \, dx \, dy. \end{aligned}$$

To see that (6) is true for general regions, we cut it into type II regions. By adding the line integrals over the boundary of these regions, we obtain

$$\int_c^d Q \, dy = \iint_R Q_x \, dA.$$