Math 185 Fall 2015, Sample Midterm 2

50 minutes, one page of notes allowed

1. (3 points each) True or False (no need for justification):
   (a) If $f(z)$ is a polynomial then its Taylor series about any point has only finitely many terms.
   (b) If $f(z) = p(z)/q(z)$ for polynomials p and q, then its Laurent series about any of its singularities has only finitely many terms.
   (c) If f is analytic in $\mathbb{C}$ except for a finite number of removable singularities, then the Taylor series of f at the origin converges in the whole complex plane.
   (d) If f is nonconstant and entire then
   $$\max_{|z|\leq 1} |f(z)| < \max_{|z|\leq 2} |f(z)|.$$
   (e) There is a nonconstant entire function such that $|f(z)| < 1 + \sqrt{|z|}$ for all $z \in \mathbb{C}$.
2. (10 points) Show that there is no function analytic in $D(0,1)$ which agrees with the function $f(x) = x^2 \sin(1/x)$ on the interval $(0,1)$.
3. (10 points) Locate and classify (as essential, removable, or poles of some order) the singularities of
   $$f(z) = \frac{e^{\pi/z}}{(z-\pi)^2}.$$
   Explain why each singularity is of the kind you state. Calculate the residues at the poles.
4. (7 points) Evaluate the integral
   $$\oint_{|z|=1} z^4 e^{2/z^2}\, dz,$$
   oriented positively.
5. (8 points) Determine the number of zeros (counting multiplicity) of
   $$f(z) = 2(z-1)^3 - e^{-z}$$
   inside the open disk $D(1,1) = \{z : |z-1| < 1\}$.