1. (3 points each) True or False (no need for justification):
   (a) If \( f(z) \) is a polynomial then its Taylor series about any point has only finitely many terms.
   (b) If \( f(z) = p(z)/q(z) \) for polynomials \( p \) and \( q \), then its Laurent series about any of its singularities has only finitely many terms.
   (c) If \( f \) is analytic in \( \mathbb{C} \) except for a finite number of removable singularities, then the Taylor series of \( f \) at the origin converges in the whole complex plane.
   (d) If \( f \) is nonconstant and entire then
   \[
   \max_{|z|\leq1} |f(z)| < \max_{|z|\leq2} |f(z)|.
   \]
   (e) There is a nonconstant entire function such that \( |f(z)| < 1 + \sqrt{|z|} \) for all \( z \in \mathbb{C} \).

2. (10 points) Show that there is no function analytic in \( D(0,1) \) which agrees with the function \( f(x) = x^2 \sin(1/x) \) on the interval \( (0,1) \).

3. (10 points) Locate and classify (as essential, removable, or poles of some order) the singularities of
   \[
   f(z) = \frac{e^{\pi/z}}{(z - \pi)^2}.
   \]
   Explain why each singularity is of the kind you state. Calculate the residues at the poles.

4. (7 points) Evaluate the integral
   \[
   \oint_{|z|=1} z^4 e^{2/z^2} \, dz,
   \]
   oriented positively.

5. (8 points) Determine the number of zeros (counting multiplicity) of
   \[
   f(z) = 2(z - 1)^3 - e^{-z}
   \]
   inside the open disk \( D(1,1) = \{ z : |z - 1| < 1 \} \).