Math 185 Fall 2015, Sample Midterm 1

50 minutes, no textbook or notes. 3-5 are 10 points each.

1. (5 points) Find all solutions to $z^{10} = |z|\overline{z}^{10}$.

2. (15 points) True or false, (no need for justification):

   (a) If f is continuous on a domain D, then f must be differentiable at at least one point in D.

   (b) If D_1 is a domain, D_2 is a domain, and $D_1 \cap D_2 \neq \varnothing$, then $D_1 \cup D_2$ is a domain.

   (c) The function
   $$f(z) = \begin{cases} \frac{1}{z} & z \neq 0 \\ 0 & z = 0 \end{cases}$$
   is entire.

   (d) $\overline{\exp(\log(\overline{z}))} = z$ for all z.

   (e) If f and f' are analytic on D and $f''(z) = 0$ for all $z \in D$ then f must be constant on D.

3. Find all points of differentiability of the function $f(z) = z + |z|^2$ defined on $\mathbb{C}$, and explain why f is differentiable at those points.

4. Suppose f is analytic on a domain D, $|f(z)| \leq 1$, and $e^{f(z)} \in \mathbb{R}$ for all $z \in D$. Show that f must be constant on D.

5. Evaluate the integral:
   $$\oint_C \frac{1}{\overline{z}^n} dz$$
   where n is an integer and C is a circle of radius π centered at the origin, oriented positively. For which n does $f(z) = \frac{1}{\overline{z}^n}$ have an antiderivative in $\mathbb{C} \setminus \{0\}$?