Math 185 Fall 2015, Midterm 2, N. Srivastava
4:10pm–5:00pm, October 30, 2015

1. True or false (no need for justification). 3 points each.
   (a) If f has an antiderivative in a domain D then f is analytic in D.
   (b) If f is analytic in a domain D then it has an antiderivative in D.
   (c) The function $f(z) = z^2/\sin(z)$ has a pole at $z = 0$.
   (d) If f and g are entire and agree on infinitely many points then they must be identical.
   (e) Every entire function has at least one zero.
   (f) There is a function $f(z)$ such that f is analytic in $\mathbb{C} \setminus \{0\}$, has a simple pole at 0, and $\oint_{|z|=1} f(z)dz = 0$.
   (g) If f and g are two branches of $\log(z)$ analytic in a domain D, then $f - g$ must be constant in D.

2. (7 points) Evaluate the integral
$$\oint_{|z-3|=1} \frac{\cos(z)}{z(z-\pi)^2} dz$$
oriented positively.

3. (7 points) Find the Taylor expansion of $f(z) = \frac{1}{z}$ at $z_0 = i + 1$. What is its radius of convergence?

4. (8 points) Evaluate the integral
$$\oint_{|z|=1} (z+1)^2 e^{3/z^2} dz,$$
oriented positively.

5. (7 points) Suppose $f : \mathbb{C} \to \mathbb{C}$ is a function with $f(1/n) = 1$ for all positive integers n and $f(i) = 2$. Show that f cannot be entire.

6. Extra credit (10 points): Assume that C is a simple closed contour. Show that
$$\oint_C \frac{1}{p(z)} dz = 0$$
whenever $p(z)$ is a polynomial of degree at least two with all zeros contained inside C.