1. True or false (no need for justification). 3 points each.
   (a) If \( f \) has an antiderivative in a domain \( D \) then \( f \) is analytic in \( D \).
   (b) If \( f \) is analytic in a domain \( D \) then it has an antiderivative in \( D \).
   (c) The function \( f(z) = z^2 / \sin(z) \) has a pole at \( z = 0 \).
   (d) If \( f \) and \( g \) are entire and agree on infinitely many points then they must be identical.
   (e) Every entire function has at least one zero.
   (f) There is a function \( f(z) \) such that \( f \) is analytic in \( \mathbb{C} \setminus \{0\} \), has a simple pole at 0, and \( \oint_{|z|=1} f(z)dz = 0 \).
   (g) If \( f \) and \( g \) are two branches of \( \log(z) \) analytic in a domain \( D \), then \( f - g \) must be constant in \( D \).

2. (7 points) Evaluate the integral
   \[
   \oint_{|z|=1} \frac{\cos(z)}{z(z - \pi)^2} dz
   \]
   oriented positively.

3. (7 points) Find the Taylor expansion of \( f(z) = \frac{1}{z} \) at \( z_0 = i + 1 \). What is its radius of convergence?

4. (8 points) Evaluate the integral
   \[
   \oint_{|z|=1} (z + 1)^2 e^{3/z^2} dz,
   \]
   oriented positively.

5. (7 points) Suppose \( f : \mathbb{C} \to \mathbb{C} \) is a function with \( f(1/n) = 1 \) for all positive integers \( n \) and \( f(i) = 2 \). Show that \( f \) cannot be entire.

6. Extra credit (10 points): Assume that \( C \) is a simple closed contour. Show that
   \[
   \oint_{C} \frac{1}{p(z)} dz = 0
   \]
   whenever \( p(z) \) is a polynomial of degree at least two with all zeros contained inside \( C \).