Math 185 Fall 2015, Midterm 1, N. Srivastava
4:10pm–5:00pm, September 23, 2015

1. (5 points) Show that
$$\left|\frac{2z-i}{2+iz}\right| = 1$$
whenever $|z| = 1$.

2. (5 points) Find all values of $(i^i)^{1/2}$.

3. (15 points) True or False. No need to explain why.
   (a) The region $\{z : |z| > 1\}$ is simply connected.
   (b) There is a branch of $\log(z-3)$ which is analytic on the open disk $D(0,1) = \{z : |z| < 1\}$.
   (c) If f is a function defined in a punctured neighborhood of z_0 and
   $$\lim_{z\to z_0} f(z) = L \neq 0$$
   then
   $$\lim_{z\to z_0} \frac{1}{f(z)} = \frac{1}{L}.$$
   (d) If $f(z)$ is differentiable at z_0 then $g(z) = \mathrm{Re}(f(z))$ must be differentiable at z_0 (where differentiability means as a function of a complex variable).
   (e) If $f(z)$ has an antiderivative $F_1(z)$ in a domain D_1 and an antiderivative $F_2(z)$ in a domain D_2, then $f(z)$ has an antiderivative in $D_1 \cup D_2$.

4. (10 points)
   (a) Show that $f(z) = |z|$ is not differentiable anywhere.
   (b) Use this to show that
   $$f(z) = \frac{z}{1+|z|}$$
   is not analytic anywhere.

5. (5 points) Suppose α is real. Show that
$$\left|\int_0^1 \cos(\alpha + it)dt\right| < 2,$$
where t is a real variable.

6. (10 points) Evaluate the integral:
$$\oint_C (\overline{z} + e^{iz})dz$$
where C is a unit circle (i.e., of radius one) centered at $z_0 = 2$, oriented positively.