Math 185-5 Fall 2015, Homework 5

Due October 2 in class

1. Brown and Churchill 53.1cf, 57.1ab.
2. Evaluate
$$\oint_{|z|=2} \frac{dz}{z^2-1},$$
oriented positively.
3. Suppose f is continuous on a (not necessarily closed) contour C. For every point z not on C, define
$$g(z) := \int_C \frac{f(s)}{s-z} ds.$$
Show that g is analytic at every $z \notin C$ and that its derivative is given by
$$g'(z) = \int_C \frac{f(s)}{(s-z)^2} ds.$$
(hint: use the definition of a derivative. Do not exchange limits and integrals without justification.)
4. Suppose D is a simply connected domain and $|f(z)-1| < 1$ for all $z \in D$. Show that
$$\oint_C \frac{f'(z)}{f(z)} = 0$$
for all closed contours C contained in D.
5. Suppose D is a simply connected domain not containing the origin. Show that there is a branch of $\log(z)$ that is analytic on D. (hint: consider an antiderivative F of $1/z$ and examine $ze^{-F(z)}$ to conclude that $e^{F(z)} = cz$ on D, for some constant c).
6. (optional) Suppose f is analytic on and inside a triangle T with the exception of one point $z_0 \in$ interior(T). Suppose f is bounded in some neighborhood of z_0. Show that $\oint_T f(z)dz = 0$. (hint: rework the proof of Cauchy-Goursat for triangles done in class).