Math 185-5 Fall 2015, Homework 2

Due September 11 in class

1. Brown and Churchill 14.1cd, 18.1, 18.7, 18.8, 18.9, 18.11, 18.13, 20.3, 20.9, 24.1bd, 24.2bc.

2. Prove that a set $S \subset \mathbb{C}$ is open iff its complement $\mathbb{C} \setminus S$ is closed.

3. Prove that if S and T are open then $S \cap T$ and $S \cup T$ are open.

   (Optional) Do these statements hold for countably many sets? That is: suppose $\{S_n\}_{n=1}^{\infty}$ are open sets. Is $\cup_{n=1}^{\infty} S_n$ open? What about $\cap_{n=1}^{\infty} S_n$? Prove or disprove.

4. Is the intersection of two domains a domain?

5. In class we saw that each point z in the complex plane can be viewed as the image of a point

   $$P(z) = (2Re(z)/(1+|z|^2), 2Im(z)/(1+|z|^2), (|z|^2-1)/(|z|^2+1))$$

   on the Riemann Sphere $S^2 \setminus \{(0,0,1)\}$ where $S^2 = \{(a,b,c) : a^2+b^2+c^2 = 1\}$. Derive a formula for the inverse mapping P^{-1} from the Riemann Sphere to the complex plane. Show that circles through the north pole $N = (0,0,1)$ are mapped to straight lines and that circles parallel to the xy-plane are mapped to circles centered at the origin.

   See Section 17 of the book for a description of the Riemann Sphere.

   (optional) Show that all circles on the Riemann sphere are mapped to either circles or lines in the complex plane.

6. Recall that in class we defined convergence of a *sequence* of complex numbers as follows: $\{z_n\}_{n=1}^{\infty}$ converges to z_0 if for every $\epsilon > 0$ there is an N such that $|z_n - z_0| < \epsilon$ for every $n \geq N$. This is denoted by:

   $$\lim_{n\to\infty} z_n = z_0.$$

   We also defined the limit of a *function* of a complex variable (see section 15 of the book). Show that

   (a) If $\lim_{z\to z_0} f(z) = w_0$ then $\lim_{n\to\infty} f(z_n) = w_0$ for every sequence $\{z_n\}_{n=1}^{\infty}$ with $z_n \neq z_0$ (in case $f(z_0)$ is not defined) converging to z_0.

   (b) If $\lim_{n\to\infty} f(z_n) = w_0$ for every sequence $\{z_n\}_{n=1}^{\infty}$ ($z_n \neq z_0$) converging to z_0 then $\lim_{z\to z_0} f(z) = w_0$.

7. Show that the function $Arg(z)$ is discontinuous at every point on the nonpositive real axis.