Math 185-5 Fall 2015, Homework 12

Due November 20 in class

1. Brown and Churchill 77.4.
2. Verify algebraically that the image of a circle $\{z : |z-a|^2 = r^2\}$ under the inversion mapping $w(z) = 1/z$ is either a circle or a line (hint: substitute $z = 1/w$, expand, and switch to Cartesian coordinates $w = u + iv$ to get the appropriate equations in u, v).
3. Using the fact that every Möbius transformation is a composition of translations, dilations, rotations, and inversions, give a short (noncomputational) proof that every Möbius transformation is a bijection from $\mathbb{C} \cup \{\infty\}$ to itself.
4. For a 2×2 matrix $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $\det(M) = ad - bc \neq 0$, let $T_M(z) = \frac{az+b}{cz+d}$. Show that

$$(T_M \circ T_N)(z) = T_{MN}(z) \quad \textit{and} \quad T_M^{-1}(z) = T_{M^{-1}}(z),$$

   i.e., composition of Möbius transformations corresponds to matrix multiplication and inversion corresponds to matrix inversion.
5. Brown and Churchill 98.12, 100.1, 100.5, 100.6, 100.7a, 100.10, 102.4, 102.5.
6. Find a Möbius transformation mapping the open half plane $H = \{z : \mathrm{Im}(e^{i\pi/4}z + 1) > 0\}$ to $D(0, 1)$. (it might help to read section 101 before doing this)
7. Show that for every open half plane H there is Möbius transformation mapping H to $D(0, 1)$ and vice versa.
8. Is there a Möbius transformation mapping the entire complex plane $\mathbb{C}$ to the open unit disk? How about the complex plane minus one point?