Math 185-5 Fall 2015, Homework 11

Due November 13 in class

1. In class we showed that $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$ by computing integrals of the function $f(z) = z^{-2}\pi\cot(\pi z)$ on certain contours with half-integral coordinates. This method is quite generic and can be used to sum many other series. Adapt the method to find the sums of the series:

   - $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^2}$, by integrating the function $f(z) = z^{-2}\frac{\pi}{\sin(\pi z)}$ on a similar contour.

   Justify each step of your method (in particular, why the relevant integrals vanish as $N \to \infty$).

2. Do parts (1)-(3) of Section 10.4 (page 141-142) of `http://math.sfsu.edu/beck/papers/complex.pdf`. Part (4) is optional but recommended.

3. Consider the rational function
$$R(z) = \frac{cz+d}{(z-a)(z-b)},$$
   $a \neq b$. Show that
$$R(z) = \frac{\mathrm{Res}(R,a)}{z-a} + \frac{\mathrm{Res}(R,b)}{z-b}.$$
   Hence show that the coefficients in a partial fraction decomposition are actually just residues. This works for polynomials of arbitrary degree with distrinct roots, and is often the quickest way to compute partial fraction decompositions, rather than solving linear equations.