Math 185-5 Fall 2015, Homework 1

Due September 4 in class

1. Brown and Churchill 2.11, 5.6, 5.7, 6.15, 6.12, 9.1, 9.6, 9.9, 9.11, 11.3, 11.7, 12.1cdf, 12.4cd., 12.10, 14.5.

2. Let z and w be complex numbers such that $z\overline{w} \neq 1$. Prove that

$$\left|\frac{w-z}{1-\overline{w}z}\right| < 1 \quad \text{if } |w| < 1 \text{ and } |z| < 1,$$

and that

$$\left|\frac{w-z}{1-\overline{w}z}\right| = 1 \quad \text{if } |w| = 1 \text{ or } |z| = 1.$$

Conclude that for any fixed w with $|w| < 1$, the function

$$F : z \mapsto \frac{w-z}{1-\overline{w}z}$$

maps the open unit disk $D(0,1)$ to itself, that it interchanges w and 0, and that it $|F(z)| = 1$ whenever $|z| = 1$.

Such mappings are called *Blaschke Factors* and are useful in a variety of contexts, some of which we will encouter in this course.

3. (optional) Prove the Cauchy-Schwartz inequality:

$$|z_1 w_1 + \ldots + z_n w_n|^2 \leq \left(\sum_{i=1}^{n} |z_i|^2\right)\left(\sum_{i=1}^{n} |w_i|^2\right),$$

where the z_i and w_i are complex numbers.