Do 4 out of 5 of the following questions (indicate which ones). Each is worth 10 points.

1. Determine whether the following series converge or diverge, using appropriate tests:
   \[
   \sum_{n=1}^{\infty} \frac{(n!)^2}{n^2 + (2n)!}, \quad \sum_{n=1}^{\infty} (-1)^n \cos(1/n), \quad \sum_{n=1}^{\infty} \frac{\log n}{n^2}.
   \]

2. Consider the approximation:
   \[
   \sqrt{1 + x} \approx 1 + \frac{x}{5}.
   \]
   Give a bound for the maximum error of this approximation when \( x \in [0, 1/2] \). Justify your reasoning.

3. There are snowboarders (\( B \)) and skiers (\( S \)) at Lake Tahoe. Their populations each year are determined by the populations the previous year, according to the formulas:
   \[
   B(n) = 2B(n-1) - S(n-1), \quad S(n) = \frac{1}{2}B(n-1) + \frac{1}{2}S(n-1).
   \]
   Suppose that initially \( B(0) = 100 \) and \( S(0) = 50 \). What will their relative proportions be after a long time? Will the total population stabilize, tend to zero, or tend to infinity as \( n \) grows? *Hint: write these equations in matrix form.*

4. Suppose that
   \[
   x^2 + y^3 = \sin(s) + \cos(t) \quad \text{and} \quad xy = s - t.
   \]
   Find \( \left( \frac{\partial x}{\partial y} \right)_s \) as a function of \( x, y, s, t \).

5. Use Lagrange multipliers to find the point on the sphere
   \[
   x^2 + y^2 + z^2 = 1
   \]
   which is closest to the point \((0, 3, 4)\).