Math 121A Spring 2015, Homework 7

Due March 13 at 10am

1. Chapter 14, Section 3: 19, 21, 22.
2. If $f(z)$ is analytic on a region R and $f(z) = \frac{dF}{dz}(z)$ on R, then F is called an *antiderivative of* f. Show that if f has an antiderivative on R, then for any contour C in R beginning at z_1 and ending at z_2:
$$\int_C f(z)dz = F(z_2) - F(z_1).$$
(hint: use the chain rule and the fundamental theorem of calculus for functions of a real variable).
Use this to evaluate
$$\int_\gamma (e^z + \sin z)dz$$
where $\gamma(t) = e^{it}, t \in [0, \pi]$ is a half-circle of radius one.
3. Use Cauchy's integral formula to evaluate the following integrals:
$$\int_C \frac{e^z + \sin z}{z} dz \quad \int_{C'}, \qquad \frac{z^2 e^z}{2z + i} dz,$$
where C is the circle $|z-2| = 3$, oriented positively, and C' is the unit circle $|z| = 1$, oriented negatively.
4. Suppose f is analytic on and inside the simple closed contour C. What is the value of
$$\frac{1}{2\pi i} \oint \frac{f(z)}{z-a} dz$$
when a lies outside C?
5. Show that if f is of the form
$$f(z) = g(z) + \frac{a_1}{z} + \frac{a_2}{z^2} + \ldots + \frac{a_n}{z^n}$$
where $g(z)$ is analytic on and inside the circle $|z| = 1$, then
$$\oint f(z)dz = 2\pi i a_1,$$
where the integral is taken in the positive direction.
6. Use Cauchy's formula to show that if f is analytic inside and on a circle $|z - a| = r$, then
$$f(a) = \frac{1}{2\pi} \int_0^{2\pi} f(a + re^{i\theta})d\theta.$$
This is known as the *mean value property*, since the right hand side is an average over the circle
7. Chapter 14, Section 4: 3, 7, 10cd. (read page 680 for the definition of a pole and the order of a pole).
8. Chapter 14, Section 5: 1.