Math 121A Spring 2015, Homework 2

Due February 6 at 10am

All sections are from Chapter 1. Using a computer to plot or calculate things is optional in questions where it is asked for, but it is a great way to get a more visceral feel for the subject.

1. Do the following series converge or diverge? Why?:

   (a) $\sum_{n=1}^{\infty} \sin(\log n)$.

   (b) $\sum_{n=1}^{\infty} \log(n \sin(1/n))$.

   (c) $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$.

2. Section 10: 4, 5, 11, 22.

3. Section 13: 9, 12, 23, 24, 28. Reading Sec 13A, B, D might help if you are confused about how to divide/substitute/etc. with series.

4. Section 14: 3, 5, 8.

5. Section 15: 2, 3, 6, 11, 23a, 28, 29.

6. Use Taylor series to approximate

   $$\int_1^3 \frac{\sin x}{x} dx$$

   to within ± 0.01 of its exact value.

7. The proof of the integral test tells us that for a nonnegative *increasing* function $f(x)$,

   $$\sum_{n=1}^{N-1} f(n) \le \int_1^N f(x)dx \le \sum_{N=2}^{N} f(n).$$

   Apply this to the function $f(x) = \log x$ (where I mean natural log) to deduce the useful inequalities

   $$e \cdot \left(\frac{N}{e}\right)^N \le N! \le e \cdot \left(\frac{N+1}{e}\right)^{N+1}.$$

Note: The slightly stronger inequality:

$$e \cdot \left(\frac{N}{e}\right)^N \leq N! \leq eN \cdot \left(\frac{N}{e}\right)^N,$$

which is what was originally written here, is also true. If you can prove that then of course that is fine too.