Qualifying Exam Syllabus

Angxiu Ni

November 8th 2018, Room 959 Evans

Committee: L. Craig Evans, Fai Ma (Mechanical Engineering), Fraydoun Rezakhanlou (Chair), John Strain (Advisor)

Major Topic: Partial Differential Equations (Analysis)

- **Laplace's Equation:** fundamental solution, mean-value formulae, properties of harmonic functions, Green's function, energy methods. (2.2)
- **Heat Equation:** fundamental solution, mean-value property, maximum principle, uniqueness, regularity, energy methods. (2.3)
- **Method of Characteristics:** derivation, boundary conditions, local solution. (3.2)
- **Introduction to Hamilton Jacobi Equations:** Euler-Lagrange Equation, Hamilton's ODE, Legendre Transformation, Hopf-Lax Formula. (3.3, except 3.3.3 'weak solutions')
- **Scalar Conservation Laws:** shocks, entropy condition, Lax-Oleinik formula, weak solutions, Riemann's problem. (3.4, except 3.4.5 'decay to N-wave in L^1')
- **Sobolev Spaces:** Holder spaces, Sobolev spaces, approximation, extensions, traces, Sobolev inequalities, compactness, Poincare's inequalities, difference quotients in $W^{1,p}$ with $p < \infty$. (5.1 - 5.8.2.a)
- **Second-order elliptic equations:** weak solutions, existence of weak solutions, regularity, maximum principles. (6, except 6.5 'eigenvalues')
- **Calculus of variations:** first variation, Euler-Lagrange equation, second variation, existence of minimizers, regularity, constraints. (8.1-8.4, except 8.1.4, 8.2.4, 8.4.3, 8.4.4 'systems')
- **Hamilton-Jacobi equations:** viscosity solutions, uniqueness, Bellman equation. (10)

References: Evans, *Partial Differential Equations*

Major Topic: Numerical Solution to Differential Equations (Applied)

- **Finite difference approximations:** truncation errors, deriving finite difference appoximations. (1)
- **Boundary value problems and elliptic equations:** consistency, stability, convergence, nonlinear equations, finite differences for Laplace's equation, fourth-order differencing, deferred corrections. (2,3)
- **The initial value problem for ODEs:** Duhamel's principle, existence and uniqueness, truncation errors, one-step errors, Taylor series methods, Runge-Kutta methods, linear multistep methods. (5)
- **Zero-stability and convergence for initial value problems:** convergence, the test problem, one-step methods, zero-stability of linear multistep methods. (6)
- **Absolute stability:** definition, stability regions for linear multistep methods, systems of ODEs, stability regions for one-step methods, B-stability. (7)
- **Stiff ODEs:** stiffness, numerical methods for stiff problems, BDF methods, Runge-Kutta-Chebyshev explicit methods. (8)
- **Diffusion equations and parabolic problems:** method of lines discretizations, stability theory, stiffness of the heat equation, convergence, Von Neumann analysis, multi-dimensional problems, the locally one-dimensional method. (9)
- **Advection equations and hyperbolic systems:** advection, method of lines discretization, the Lax-Wendroff method, upwind methods, Von Neumann analysis, characteristic tracing, the CFL condition, modified equations, hyperbolic systems, initial boundary value problems. (10)

References: LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations*

Minor Topic: Dynamical Systems (Analysis)

- **Ergodic theory:** invariant measures, von Neumann ergodic theorem, Birkhoff ergodic theorem, mixing (1)
- **Transfer operator:** basic properties, applications to expansion on [0,1], baker's transformation, Liouville's equation. (2)
- **Lyapunov Exponents:** definitions, examples, Oseledets theorem (Chapter 4, till before theorem 4.4 '$h_\mu \leq \sum l_j^+$')

Reference: Rezakhanlou, *Lectures on Dynamical Systems*