Qualifying Exam Syllabus

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Major Topic: Partial Differential Equations (Analysis)

- **Laplace’s Equation**: fundamental solution, mean-value formulae, properties of harmonic functions, Green’s function, energy methods. (2.2)

- **Heat Equation**: fundamental solution, mean-value property, maximum principle, uniqueness, regularity, energy methods. (2.3)

- **Method of Characteristics**: derivation, boundary conditions, local solution. (3.2)

- **Introduction to Hamilton Jacobi Equations**: Euler-Lagrange Equation, Hamilton’s ODE, Legendre Transformation, Hopf-Lax Formula. (3.3, except 3.3.3 ‘weak solutions’)

- **Scalar Conservation Laws**: shocks, entropy condition, Lax-Oleinik formula, weak solutions, Riemann’s problem. (3.4, except 3.4.5 ‘decay to N-wave in $L^1$’)

- **Sobolev Spaces**: Holder spaces, Sobolev spaces, approximation, extensions, traces, Sobolev inequalities, compactness, Poincare’s inequalities, difference quotients in $W^{1,p}$ with $p < \infty$. (5.1 - 5.8.2.a)

- **Second-order elliptic equations**: weak solutions, existence of weak solutions, regularity, maximum principles. (6, except 6.5 ‘eigenvalues’)

- **Calculus of variations**: first variation, Euler-Lagrange equation, second variation, existence of minimizers, regularity, constraints. (8.1-8.4, except 8.1.4, 8.2.4, 8.4.3, 8.4.4 ‘systems’)

- **Hamilton-Jacobi equations**: viscosity solutions, uniqueness, Bellman equation. (10)

References: Evans, *Partial Differential Equations*
Major Topic: Numerical Solution to Differential Equations (Applied)

- **Finite difference approximations**: truncation errors, deriving finite difference approximations. (1)

- **Boundary value problems and elliptic equations**: consistency, stability, convergence, nonlinear equations, finite differences for Laplace’s equation, fourth-order differencing, deferred corrections. (2,3)

- **The initial value problem for ODEs**: Duhamel’s principle, existence and uniqueness, truncation errors, one-step errors, Taylor series methods, Runge-Kutta methods, linear multistep methods. (5)

- **Zero-stability and convergence for initial value problems**: convergence, the test problem, one-step methods, zero-stability of linear multistep methods. (6)

- **Absolute stability**: definition, stability regions for linear multistep methods, systems of ODEs, stability regions for one-step methods, B-stability. (7)

- **Stiff ODEs**: stiffness, numerical methods for stiff problems, BDF methods, Runge-Kutta-Chebyshev explicit methods. (8)

- **Diffusion equations and parabolic problems**: method of lines discretizations, stability theory, stiffness of the heat equation, convergence, Von Neumann analysis, multidimensional problems, the locally one-dimensional method. (9)

- **Advection equations and hyperbolic systems**: advection, method of lines discretization, the Lax-Wendroff method, upwind methods, Von Neumann analysis, characteristic tracing, the CFL condition, modified equations, hyperbolic systems, initial boundary value problems. (10)

References: LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations*

Minor Topic: Dynamical Systems (Analysis)

- **Ergodic theory**: invariant measures, von Neumann ergodic theorem, Birkhoff ergodic theorem, mixing (1)

- **Transfer operator**: basic properties, applications to expansion on [0,1], baker’s transformation, Liouville’s equation. (2)

- **Lyapunov Exponents**: definitions, examples, Oseledets theorem (Chapter 4, till before theorem 4.4 ‘\( h_{\mu} \leq \sum l_j^+ \)’)

Reference: Rezakhanlou, *Lectures on Dynamical Systems*