MATH 54 REVIEW SESSION OCTOBER 26, 2017

CHAPTERS 4.1 - 6.5

4.1 vector spaces, subspaces

* know the axioms of each *
  - what are they about?
  - conceptually understand

subspaces are vectors spaces

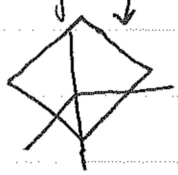
NON EXAMPLES

solution set of $A\underline{x} = \underline{b}$ for $b \neq 0$

may look like a subspace, but it would (might) be displaced from the origin.

$H + v$

Null(A) a solution



EXAMPLES

vector spaces

① $\mathbb{R}^n$, $n \geq 0$

② $S = \mathbb{R}^\infty \{a_1, a_2 \dots\}$

③ $S_0 = \{(a_1, a_2 \dots)$ eventually zero$\}$

④ functions $\{f: \mathbb{R} \to \mathbb{R}\}$

⑤ continuous functions $\{f: \mathbb{R} \to \mathbb{R}$ cont$\}$

⑥ $P = \{f: \mathbb{R} \to \mathbb{R}$ polynomial$\}$ ← infinite

⑦ $P_{\leq n} = \{f: \mathbb{R} \to \mathbb{R}$ polynomial of deg. $\leq n\}$ ↙ $n+1$

subspaces

① Span of a list of vectors $\{v_1 \dots v_k\} \in V$

② Null(T) $\in V$, $T: V \to W$ (linear transformation)

③ Image(T) $\in W$, $T: V \to W$ (Image(A) = Col(A))

[axioms of subspace $H \subset V$]

- $\underline{0} \in H$
- $\underline{u}, \underline{v} \in H \to \underline{u} + \underline{v} \in H$
- $\underline{u} \in H \to c\underline{u} \in H$

$\boxed{4.2}$ NULL SPACES, COL SPACES, LIN. TRANSFORMATIONS

* know axioms of linear transformations *

be comfortable working with abstract V, W and abstract T

$\Downarrow$

$T: P_{\leq 3} \to P_{\leq 4}$

$T(p(x)) = xp(x) + p'(z) \ldots$

EXAMPLE

m x n matrix defines a lin. transformation (LT) $T: \mathbb{R}^n \to \mathbb{R}^n$ by acting on vectors.

$\boxed{4.3}$ LINEARLY INDEPENDENT SETS & BASES

* be familiar w/ the definitions *

$v_1 \ldots v_k$ are linearly independent $(\mathbb{R}^n)$	vs. $v_1 \ldots v_k$ span $\mathbb{R}^n$
$k \leq n$	$k \geq n$
removing vectors ~~[illegible]~~ can't hurt	adding vectors can't ~~[illegible]~~ hurt
$n\{[\quad]$ with $\underbrace{\quad}_{k}$ pivot in each column	$[\quad]$ pivot in each row
A is "injective"	A is "surjective"
Null(A)=0	Image(A) = $\mathbb{R}^n$

[BASES → linearly independent & spans.]

- if $v_1 \ldots v_k$ is linearly independent in $\mathbb{R}^n$, we can always find $v_{k+1} \ldots v_n$ so that $v_1 \ldots v_n$ is a basis.
- if $v_1 \ldots v_k$ spans $\mathbb{R}^n$, then you can always remove vectors so that remaining vectors form a basis. (take pivot columns of the original matrix).
- a list $v_1 \ldots v_k$ is linearly ~~in~~dependent

if & only if... → iff some v_i is in the span of the preceeding ones,.. first free column

- single vector linearly independent? not zero.

4.5 & 4.6

DIMENSION & RANK

Any two bases of V have the same # of vectors.

[dim V = size of the basis]

EXAMPLE

$$\text{Null}\left(\begin{vmatrix} 1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 3 \end{vmatrix}\right) \subset \mathbb{R}^4$$

but the dim of the Null is given by the number of free columns, which is 2.

{* know how to find the null * space and image}

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 3 \end{vmatrix}$$

Basis of Null

$$B = \left\{ \begin{vmatrix} -1 \\ 0 \\ 1 \\ 0 \end{vmatrix}, \begin{vmatrix} 0 \\ 3 \\ 0 \\ 1 \end{vmatrix} \right\}$$

how many vectors? thats the dim of null space, not the number of #s.

~~[illegible]~~

understand the relationship of rank and nullity to matrix algebra !

{ A, B are m×n
rank A = a
rank B = b
rank(A+B) → what are the possibilities?
anywhere b/t ~~[illegible]~~ (0 ... a+b)
↑ or min(a+b, n) }

understand conceptually ↓

spans → apply surjective → still spans

linearly ind → injective → still linearly independent

- dim H ≤ dim V
  H ⊂ V subspace

rank of T = dim image(T)

RANK THM.

dim V is finite.

T : V → W is a linear transformation.

dim V = dim Null(T) + rank(T)

{for matrices, # free cols + # pivot cols}

→ also: what is the possible rank of (AB)?

- the image can't get bigger
- A, m×n, B, m×k,
  rank(A) = a
  rank(B) = b
  rank(AB) ≤ min{a, b}

4.4 & 4.7

COORDS

[T_β^{-1} is usually easier to find]

if I'm given a basis $\beta = \{\underline{v}_1 \dots \underline{v}_n\}$ of V

$T_\beta : V \to \mathbb{R}^n$ this is invertible. what is the inverse map? $V \leftarrow \mathbb{R}^n : T_\beta^{-1}$

$T_\beta(\underline{v}) = \begin{vmatrix} a_1 \\ \vdots \\ a_n \end{vmatrix}$ so that $\underline{v} = a_1\underline{v}_1 + \dots a_n\underline{v}_n$ $\quad T_\beta^{-1}\left(\begin{vmatrix} a_1 \\ \vdots \\ a_n \end{vmatrix}\right) = a_1\underline{v}_1 + \dots a_n\underline{v}_n$

you should be able to calculate T_β, T_β^{-1} in examples $\to P_{\leq n}$, $H \subset \mathbb{R}^n$

T_β: solve a linear system

T_β^{-1}: multiply & add

$\left\{ \begin{matrix} P_\beta = T_\beta^{-1} \\ P_\beta^{-1} = T_\beta \end{matrix} \right\}$

$T_\beta : \mathbb{R}^n \to \mathbb{R}^n$

$\|$

$\underset{\beta \leftarrow \mathcal{E}}{P}$

$\downarrow$

$\underset{\mathcal{E} \leftarrow \beta}{P^{-1}}$

$\downarrow$

P_β^{-1}