University of California, Berkeley
Fall 2021, Math 215A
Midterm 2

Instructor: Prof. David Nadler
GSI: Ethan Dlugie

November 1 to 5, 2021

Instructions

You may work on this exam at any point between 9:00AM on November 1, 2021 and 11:59PM on November 5, 2021. Before the November 5 deadline, complete and upload your solutions to the three problems below on Gradescope.

You are permitted to use your textbook, your notes, and any other resources that you have produced or that the instructor has provided as part of this course. No external material is allowed.

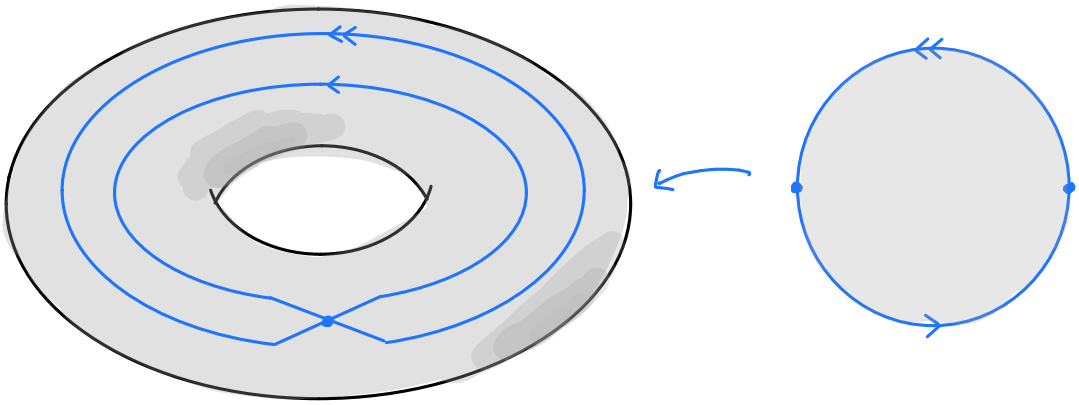
You are not permitted to discuss these problems with your fellow classmates or anyone else until the solutions have been posted. All work must be your own.

Email the instructor (Professor Nadler) or GSI (Ethan Dlugie) if you have any issues. Otherwise, good luck!

Questions

1. Let $F_2 = \langle a, b \rangle$ be the free group on two generators. Give an explicit example of a finite index, normal subgroup H of F_2 that does not contain the element ab^2. (By "explicit", we mean give a generating set for H.) Of course, you should prove that your H satisfies the requirements.

2. Consider a space Z constructed by gluing the boundary of a disk along the curve shown in the torus:



   Compute the homology groups of Z.

3. Let $p : \mathbb{R} \to \mathbb{R}$ be a polynomial with real coefficients. Show that p can always be extended to a continuous map of one-point compactifications $\hat{p} : S^1 \to S^1$. Find a formula for the degree of $\hat{p}$ in terms of the polynomial p.