PERVERSE SHEAVES ON CURVES

X smooth curve \rightarrow \text{Perf}(X) abelian category \subseteq D^b(X) derived cat.
of sheaves on X

In fact, \text{Perf}(X) \subseteq D^b(X) = derived cat. not abelian, but
of constructible \textit{truncated} \varepsilon

Get

A complex \mathcal{F} on X is constructible if there is an open \mathcal{U} \subseteq X,
closed \mathcal{Y} = \mathcal{X} \setminus \mathcal{U} s.t. \mathcal{H}^n\mathcal{F}_{\mathcal{U}} \text{ is locally constant with } \mathcal{O}_\mathcal{F}
\text{stalks}

\mathcal{H}^n\text{ bordaries finitely summands with restriction to } \mathcal{U}

and \mathcal{H}^n\mathcal{F} = \mathcal{O}_\mathcal{F}.

\text{i.e. can cut up } X \text{ into pieces and } \mathcal{H}^n\mathcal{F} \text{ doesn't jump between pieces}

Ex.

X = \mathcal{P}^1

1) Connect that \mathcal{O}_{\mathcal{P}^1} \text{ is twisted by not.}

2) Hopf fibration \mathcal{S}^2 \rightarrow \mathcal{P}^1 (or any circle bundle)

\mathcal{R} \rightarrow \mathcal{P} \rightarrow \mathcal{P} \text{ pull back in derived cat.}

3) \mathcal{H}^0 = 0, \mathcal{H}^1 = \mathcal{O}_{\mathcal{P}^1} \otimes \mathcal{E}

4) \mathcal{H}^2 = \mathcal{E}^2 \otimes \mathcal{E}

5) \mathcal{H}^3 = 0

\mathcal{H}^n = \mathcal{O}_{\mathcal{P}^1} \otimes (c_n \mathcal{O}_{\mathcal{P}^1}) \rightarrow \mathcal{H}^n = 0

\text{contains an annulus relative to } \mathcal{D}

\text{Perf}(X) = D^b(X) \text{ consists of } \mathcal{F} \text{ s.t. for which } u \to \tilde{u} \text{ open, } Y_{\tilde{u}} \subseteq Y_u
\text{ in which } \mathcal{F} \text{ is constructible, we have } \mathcal{H}^n\mathcal{F}_{\tilde{u}} \text{ is concentrated in degree } -1
\text{ and } \mathcal{H}^n\mathcal{F}(\tilde{u}) \text{ conc. in degree } -1, 0

\text{Thus that } \text{Perf}(X) \text{ is actually abelian.}
\[ i^* = "\text{sections near } Y" \]
\[ i^! = "\text{sections supp on } Y \text{ (zoom off } Y)" \]

5. Exercise

\[ X = P^1, \ Y = \text{int } Y, \ u = \text{aff } \]

1) \( C^3_{\text{aff}} \cong H^0(\mathcal{F}) \) has a deg 2 section, but we shifted down to deg 1, so \( H^0(\mathcal{F}) \) con. in deg 2.

2) \( C^3_{\text{aff}} \) is a translate of \( Y = C^3_{\text{int}} \), so con. in deg 1.

3) \( \text{int } C^3_{\text{aff}} \) is \( C^3_{\text{int}} \) (coming out of boundary) \( \mathcal{F} \) \( \cong 0 \text{ sheaf } \).

4) \( j^! C^3_{\text{aff}} \) \( \cong j^! C^3_{\text{int}} \) \( \cong C^3_{\text{aff}} \), since it's inside \( Y \).

5) Exercise: Check \( T \) is perverse.

A decomposable under Riemann-Hilbert.

Back to our description, now for \( 0 \leq t \leq 0 \delta \).

Then \( \text{Per} \circ (0, 0) = \mathbb{C} \) modules over \( \mathcal{O}_X \).

From this, we can see why \( \text{Per} \circ (0, 0) \) is an abelian cat, since it is just modules.

Construction of functor \( \text{Per} : \mathbb{C}^{\text{diff}}(\mathcal{O}_{X}) \to \mathbb{C} \).

\( i^{\text{aff}} \) is sections on \( \mathcal{F} \) on \( \mathcal{F} \) vanishing on \( C^3_{\text{aff}} \).

The relative \( \text{Per} \) is \( \mathcal{F} \).