

Name: _____

Math 256A Quiz 8

Due Thursday 11/14

1. (a) Show if X is a smooth group-scheme, then the tangent bundle TX is trivializable.
(b) Show if X is a smooth connected projective group-scheme, then X is abelian.
(c) Classify all smooth connected projective one-dimensional group-schemes.
2. Suppose L_1, L_2 are line bundles on a smooth projective curve C of respective degrees ℓ_1, ℓ_2 .
(a) Show if $\ell_1 > \ell_2$, then the only map of line bundles $L_1 \rightarrow L_2$ is the zero map.
(b) Show by example if $\ell_1 < \ell_2$, the only map of line bundles $L_1 \rightarrow L_2$ may be the zero map.
3. (a) Show every line bundle on \mathbb{P}^1 is isomorphic to one of the form $\mathcal{O}_{\mathbb{P}^1}(n)$, $n \in \mathbb{Z}$.
(b) Show every vector bundle on \mathbb{P}^1 is isomorphic to a sum of line bundles.