1. For each of the listed graded rings R, describe $\operatorname{Proj} R$. We write $|r|$ for the degree of an element $r \in R$.
   (a) $R = \mathbb{C}[x, y]/(x^2, y^2)$ with $|x| = |y| = 1$.
   (b) $R = \mathbb{C}[w, x, y, z]/(wz - xy, wy - x^2, xz - y^2)$ with $|w| = |x| = |y| = |z| = 1$.
   (c) $R = \mathbb{C}[x, y]$ with $|x| = 1, |y| = 2$.
   (d) (challenge) $R = \mathbb{C}[x, y, z]$ with $|x| = 1, |y| = 2, |z| = 3$.

2. Given a scheme Y, recall each point $y \in Y$ is the image of a map $y : \operatorname{Spec} k(y) \to Y$, where $k(y) = \mathcal{O}_{Y,y}/\mathfrak{m}_y$ with $\mathcal{O}_{Y,y}$ the stalk of the structure sheaf of Y at y, and $\mathfrak{m}_y \subset \mathcal{O}_{Y,y}$ the maximal ideal. (Locally, if we have $Y = \operatorname{Spec} R$ and $y = \mathfrak{p}$, then $\mathcal{O}_{Y,y} = R_{\mathfrak{p}}$, and $\mathfrak{m}_y = R_{\mathfrak{p}}\mathfrak{p}$.) Given a map of schemes $f : X \to Y$, and a point $y \in Y$, define the fiber to be the fiber product $X \times_Y y$.

   Calculate the fiber of the map $f : \mathbb{A}^2 \to \mathbb{A}^2$, $f(x, y) = (x, xy)$ at the listed points.
   (a) any closed point (a, b) given by a maximal ideal $(x - a, y - b) \subset \mathbb{C}[x, y]$.
   (b) the generic point given by the zero ideal $(0) \subset \mathbb{C}[x, y]$.
   (c) a (neither closed nor open) point given by a prime ideal $(p) \subset \mathbb{C}[x, y]$ for irreducible $p \neq 0$.

3. Let X be an irreducible, reduced scheme with generic point x and associated field $k(x)$.
   (a) Describe maps $X \to \mathbb{P}^1$ in terms of elements of $k(x)$.
   (b) Describe all scheme maps $\mathbb{P}^2 \to \mathbb{P}^1$.