Name: ______________ Math 256A Quiz 4 Due Thursday 10/3

1. For each of the following integral domains $R$, find its integral closure $R' \supset R$, and describe the map $\operatorname{Spec} R' \to \operatorname{Spec} R$.
   (a) $R = \mathbb{C}[x, y]/(x^3 - y^2)$
   (b) $R = \mathbb{C}[x, y]/(x^2 - x^3 + y^2)$
2. Describe the map $\operatorname{Spec} \mathbb{C}[x, y] \to \operatorname{Spec} \mathbb{R}[x, y]$ of locally ringed spaces – in particular, describe the underlying map of spaces and the induced maps on stalks of structure sheaves – induced by the inclusion $\mathbb{R}[x, y] \subset \mathbb{C}[x, y]$.
3. Let $R$ be a commutative $\mathbb{C}$-algebra. Show that to give a map $\operatorname{Spec} \mathbb{C}[\epsilon]/(\epsilon^2) \to \operatorname{Spec} R$ over $\operatorname{Spec} \mathbb{C}$ is the same as to give a point $\mathfrak{p} \in \operatorname{Spec} R$ such that $\mathbb{C} \simeq R/\mathfrak{p}$, along with an element of the dual $\mathbb{C}$-vector space $(\mathfrak{p}/\mathfrak{p}^2)^*$.